University of the Highlands and Islands
Perth College

National 5 Maths

Conditional offer workbook

Applicant name : ______________________________

Please complete this workbook within one calendar month of receipt.

Place the workbook in an A4 envelope and post it first class recorded mail (cost £2·84 at time of writing) to

FAO G Fyfe
Core Skills dept
Perth College
Crieff Road
Perth
PH1 2NX

Please note a normal stamp on your envelope is not enough! Underpaid postage may lead to a delay in your application and your envelope may even be refused.

Please also enclose a ssae if you would like your workbook returned (see FAQ).

If you live locally you can just hand it in personally at the main college reception.

FAQ

- Why do I have to do this workbook when I already have the necessary qualification given on your website/prospectus?

  We get a huge number of applications for National 5. We only give out <u>un</u>conditional offers to applicants who are resitting, or applicants who have just achieved their background qualification the previous summer.

  It is very easy to forget many of the background skills if it has been a while since you studied maths.

  Doing this workbook will prepare you very well for National 5, and increase your chances of coping with what can be a surprisingly demanding course.

- What happens when I send my workbook back?

  It is carefully marked by one of the maths tutors. You then receive written feedback by email on your answers. Depending on how you have done, you will either be offered either an unconditional place or your application will be rejected.

- What is the pass mark?

  We go by the overall standard of your answers.

  You probably won't get everything perfectly correct, and there may be topics you struggle with. However almost everybody does a good enough job to get on the course. Only about 4% of workbooks returned are not of a suitable standard.

  Do your best and enjoy getting back into maths!

- Can I have my book back when it has been marked?

  Yes we can return it to you, but only if you enclose a suitable stamped, self-addressed envelope when you send it in.

  If you live locally you can collect your marked workbook from the college by arrangement.

- Something has happened and I don't think I will be able to return the workbook within one month. What should I do?

  Please contact us immediately. Places on the course are limited, and if you don't tell us you risk losing your place.

Introduction

We are pleased that you want to study National 5 maths. Maybe you need it to enter teacher training. Perhaps you want to move into nursing or midwifery. Maybe you hope to join the RAF or go into social work. Or maybe you just want to improve your maths.

In any case you want to pass, so it is important you know what to expect and that you are well prepared for starting the course.

Here is a list of the current maths courses offered in Scotland:

National 3	Pretty straightforward, most people could probably cope with this without special preparation. Old equivalents were Access 3 maths and Foundation level maths (standard grade 5 or 6).
National 4	Starts getting more mathematical, for example contains equations, Pythagoras, π and Trigonometry. Old equivalents were Intermediate 1 maths and General level maths (standard grade 3 or 4). This is the normal entry requirement for National 5. National 4 Lifeskills is a different course which is not suitable for progression.
National 5	Much more mathematical. Most of the general public would only understand a little of this. Old equivalents were O-Grade maths, Intermediate 2 maths and Credit level maths (standard grade 1 or 2). This is the usual entry requirement for the careers mentioned earlier.
Higher	Proper specialised maths. Most people would probably just shake their heads at this, which is a pity because this is where it starts to get really interesting!
Advanced Higher	Absolute hieroglyphics, and absolutely fascinating. Old equivalent was CSYS. English A-level is similar.

The point of this workbook is to help you brush up on the most important parts of the National 4 course – the parts that will help you do well at National 5.

For European applicants

In Britain we write some mathematical things differently to the way you write them in Europe.

$\boxed{,}$

Commas can be confusing. In Europe, a comma separates the units from the tenths; the whole number from the decimal part. For example, you maybe weigh 70,2 kg.

In Britain, commas are sometimes used to make large numbers easier to read. They come after the thousands or the millions. For example, the population of Perth is 50,000 and the number of people living in Scotland was 5,295,403 according to the 2011 census.
Using commas like that is a bit old-fashioned. We don’t use them like that in this workbook, we leave a small space instead.

$\boxed{\cdot}$

In Europe, this means multiply. For example 2·7 means 14

In Britain, it is a decimal point. So 2·7 here means the same as the European 2,7

When we write $2\cdot5^2$ it means $2{,}5^2 = 2{,}5 \cdot 2{,}5 = 6{,}25$ and does **not** mean $2 \cdot 25 = 50$!
If we want to show we are multiplying the 2 by the 7, we write 2×7 instead.

$\boxed{:}$

In Europe, 20 : 4 means divide and get the answer 5.
We do use the : sign in Britain, but only in things called ratios which **compare** two amounts. For example, the ratio of men to women might be 5 : 6. That means there are five men for every six women, it’s not a sum where you need to do anything.

In Britain if we want to divide we use a divide sign $\div$ instead, so we write $20 \div 4 = 5$

Calculator

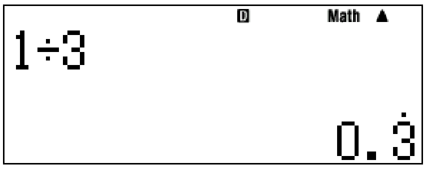
This is a Casio fx-85 calculator.

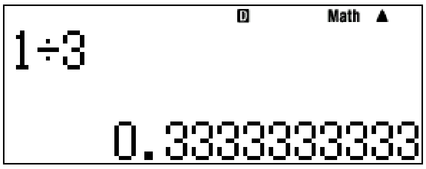
You will need one (an fx-82 or 83 is almost as good) for National 5 maths and for some of the work in this book. There are other makes of good calculators, but they don't all have the special features the fx-85 possesses.

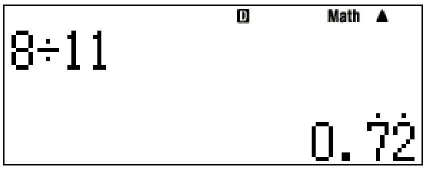
If you have not used one of these before, please read through the owner's manual.

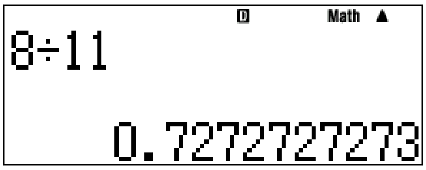
Before you go any further in this book, make sure you know

- how to save time using the cursor, the delete [DEL] and last answer [Ans] buttons.
- how to change the shape of "weird" answers using the [S⇔D] button.
- how to change between Line mode and Math mode.
- how to use the fraction button $\left[\frac{\square}{\square}\right]$.
- that $0\cdot\dot{3}$ means endless 3s and $0\cdot\dot{7}\dot{2}$ means 0·72727272 … repeating endlessly.
- the button for entering negative numbers is [(−)], not [−].
- how to enter π - it's in yellow writing above your [$\times10^x$] button.









(The "3" at the end here is just the calculator rounding things off.)

Contents

Rounding off

1) We round numbers off all the time. If a sports report says that there were 6 000 people at a football match it is highly unlikely there were **exactly** 6 000. The exact figure may have been 6 207 or 5 876, it doesn't really matter, but we do know the precise figure was closer to 6 000 than it was to 5 000 or 7 000.

The attendance has been rounded to the nearest thousand.

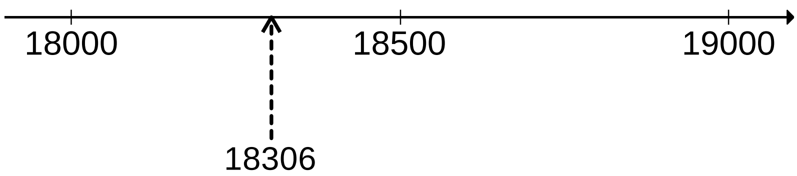
Numbers which have been rounded to the nearest thousand always end in the figures "000", for example 6 000 or 73 000.

2) Suppose we want to round 18 306 off to the nearest thousand. Let's show our intentions using this notation with an arrow:

$$18306 \xrightarrow{1000}$$

Because the number is 18 000-and-a-bit, the answer will either be 18 000 or 19 000. It all depends whether our number is more or less than halfway, and halfway between 18 000 and 19 000 is of course ________

Here's a picture of the situation.



Because the 18 306 is less than 18 500, our number is in the left-hand portion and is therefore closer to 18 000 than 19 000. We'd write

$$18\,306 \xrightarrow{1000} 18\,000$$

Round these off to the nearest thousand:

a) $4\,277 \xrightarrow{1000}$

b) $24\,860 \xrightarrow{1000}$

c) $9\,895 \xrightarrow{1000}$

d) $67\,467 \xrightarrow{1000}$

3) What if the number already happens to be exactly halfway, such as 6 500? It is equidistant from 6 000 and 7 000. The convention nowadays is to always round up in this situation.

a) $8\,500 \xrightarrow{1000}$

b) $51500 \xrightarrow{1000}$

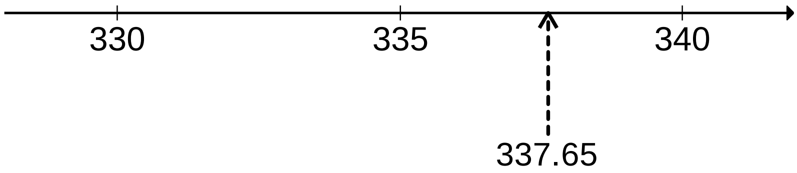
4) It is just as easy to round off to the nearest hundred, ten or unit.

Hundreds like 400 or 1 200 end in “00”. The halfway number for a hundred is 50

Tens like 30 or 180 end in “0”. The halfway number for a ten is ____

Units like 19 or 5 are just plain whole numbers. The halfway number for a unit is ____

For example, to round 337·65 to the nearest ten, we have to choose between 330 and 340.



It doesn’t matter that the number is a decimal, we just need to know that 337·65 is greater than the halfway number, 335, so we round up.

$$337\cdot 65 \xrightarrow{10} 340$$

Round these off as asked

a) $4\,489 \xrightarrow{10}$

b) $3\,333\cdot 3\,333 \xrightarrow{100}$

c) $28\cdot 625 \xrightarrow{\text{unit}}$

d) $1414 \xrightarrow{10}$

e) $95\cdot 2 \xrightarrow{\text{unit}}$

f) $563\cdot 5625 \xrightarrow{10}$

g) $33\,825\cdot 15 \xrightarrow{100}$

h) $17\cdot 795 \xrightarrow{\text{unit}}$

i) $6\,786\cdot 286\,28 \xrightarrow{100}$

j) $13\cdot 499 \xrightarrow{\text{unit}}$

5) We have seen how to round to the nearest 1000, nearest unit and so on. But often in maths we require greater accuracy, and this means leaving answers as decimals.

- Decimals like 4·1 and 19·0 are said to have one decimal place or 1 dp for short
- Decimals like 0·83 and 122·87 are said to have two decimal places or 2 dp for short
- Decimals like 14·200 and 0·625 are said to have three decimal places or 3 dp for short

6) Type in the sum $41 \div 7 =$ _______________

This decimal goes on for ever and there are various ways of rounding it off depending on how accurate we want to be.

$$41 \div 7 = 5 \cdot 857142857$$
$$\xrightarrow{\text{unit}} 6$$
$$\xrightarrow{\text{1 dp}} 5 \cdot 9$$
$$\xrightarrow{\text{2 dp}} 5 \cdot 86$$
$$\xrightarrow{\text{3 dp}} 5 \cdot 857$$

If we want to round to 1 dp, we need to look at the second figure after the point.
Here it's a "5". Halfway or more, round up.

For 2 dp, we look at the third figure after the point.
Here it's a "7". Halfway or more, round up.

For 3 dp, look at the _______ figure after the point. Here it's only a "___".
Less than 5, round down.

Remember, always look at the **next** figure when rounding to decimal places.
You may find it helpful to draw in the "cut" line and circle the next figure.

7) Round these off as asked:

a) $3 \cdot 48095231 \xrightarrow{\text{1 dp}}$ ✂

b) $41 \cdot 314 \xrightarrow{\text{2 dp}}$ ✂

c) $7 \cdot 62 \xrightarrow{\text{1 dp}}$

d) $0 \cdot 87 \xrightarrow{\text{1 dp}}$

e) $84 \cdot 295 \xrightarrow{\text{1 dp}}$

f) $3 \cdot 634 \xrightarrow{\text{2 dp}}$

g) $22 \cdot 5454 \xrightarrow{\text{2 dp}}$

h) $103 \cdot 4169 \xrightarrow{\text{2 dp}}$

i) $0 \cdot 0671 \xrightarrow{\text{2 dp}}$

j) $8 \cdot 806377 \xrightarrow{\text{3 dp}}$

k) $474 \cdot 163902 \xrightarrow{\text{3 dp}}$

l) $5 \cdot 26499 \xrightarrow{\text{2 dp}}$

8) Sometimes you get a "knock-on effect" when rounding off. Suppose we want to round 3·96 to one decimal place. We can either leave it as 3·9 or round up to … well, what **is** the next choice!?

The next number after 9 is of course 10, but we can't have 3·10 because that's much smaller than 3·9. The column containing the "9" is already full. So it must spill over into the Units column. The next number is 4·0 (It's a bit like saying the next number after 39 is 40).

Note the answer "4" would be marked incorrect. The answer must have one decimal place.

a) $7\cdot 99 \xrightarrow{\text{1 dp}}$

b) $7\cdot 8984 \xrightarrow{\text{2 dp}}$

c) $28\cdot 3197 \xrightarrow{\text{3 dp}}$

d) $18\cdot 999555 \xrightarrow{\text{3 dp}}$

9) We often get decimals as the answers to sums on our calculators. Type in the following sums, write down the decimals and round the answers off as asked.

a) $\frac{2}{3} = 0\cdot 66666666...$

$\xrightarrow{\text{3 dp}}$

b) $\frac{645}{13} =$

$\xrightarrow{\text{2 dp}}$

c) $0\cdot 72 \times 0\cdot 43 =$

$\xrightarrow{\text{3 dp}}$

d) $\sqrt{20} =$

$\xrightarrow{\text{2 dp}}$

e) $\frac{5}{11} =$

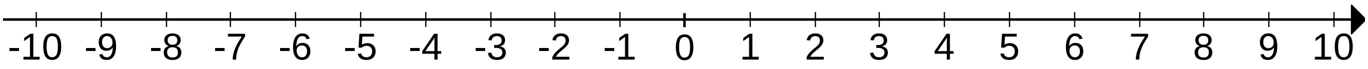
$\xrightarrow{\text{3 dp}}$

f) $\sqrt{440} =$

$\xrightarrow{\text{1 dp}}$

Negative numbers

10) Here is a number line in case you need it



What number do we arrive at if we start at

a) 3 and go down 4?

b) 6 and go down 10?

c) 4 and go down 6?

d) 0 and go down 5?

11) What number do we arrive at if we start at

a) −2 and go down 3?

b) −5 and go down 10?

c) −1 and go down 6?

d) −7 and go down 2?

12) Going down!

a) $5-8=$

b) $1-7=$

c) $-5-2=$

d) $4-10=$

e) $-3-7=$

f) $-1-9=$

g) $-2-1=$

h) $2-4=$

i) $3-7=$

j) $-4-3=$

k) $3-6=$

l) $4-11=$

13) What number do we arrive at if we start at

a) −5 and go up 2?

b) −3 and go up 5?

c) −2 and go up 1?

d) −2 and go up 8?

e) −10 and go up 3?

f) −2 and go up 5?

g) −8 and go up 3?

h) −4 and go up 4?

14) Going up!

a) $-6+1=$

b) $-1+6=$

c) $-3+1=$

d) $-10+2=$

e) $-2+9=$

f) $-8+1=$

g) $-7+11=$

h) $-4+1=$

i) $-5+13=$

j) $-5+12=$

k) $-6+11=$

l) $-8+6=$

15) A mixture of "ups" and "downs":

a) $3-11=$

b) $-2+8=$

c) $1-7=$

d) $-7+3=$

e) $-8-2=$

f) $-8+2=$

g) $-1-7=$

h) $2-9=$

i) $-9+1=$

16) In maths you are not allowed to have two signs sitting next to one another.
So we cannot write sums like

$$4+-8 \qquad \text{or} \qquad 6--5$$

Instead we must wrap the negative number in brackets to keep the signs apart:

$$4+(-8) \qquad \text{or} \qquad 6-(-5)$$

But why have two signs if you can replace them with one single sign?! The sum $4+(-8)$ means we start at 4 and either go up 8 or go down 8. So we can rewrite $4+(-8)$ as one of these:

$$\begin{aligned} 4+(-8) &= 4+8 \\ &= 12 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 4+(-8) &= 4-8 \\ &= -4 \end{aligned}$$

Which one do you think is correct?

Good, it's the second one.

When the two signs separated by the bracket are different they can be replaced by a single "**–**" sign.

Likewise

$$-3+(-7)=-3-7$$
$$=$$

Conversely when the two signs separated by the bracket are the same they can be replaced by a single "**+**" sign.

$$6-(-5)=6+5$$
$$=$$

When the two signs are **differen**t they can be replaced by a single "**−**" sign. When the two signs are the **same** they can be replaced by a single "**+**" sign.

17) Rewrite these sums with a single sign between the numbers and work out the answers:

a) $10+(-9)=10\quad 9$
$=$

b) $-10+(-4)=-10\quad 4$
$=$

c) $-3+(-4)=-3\quad 4$
$=$

d) $-9+(-1)=-9\quad 1$
$=$

e) $5-(-8)=5\quad 8$
$=$

f) $0+(-8)=0\quad 8$
$=$

g) $5+(-8)=5\quad 8$
$=$

h) $9-(-3)=9\quad 3$
$=$

i) $-7+(-1)=-7\quad 1$
$=$

j) $5+(-5)=5\quad 5$
$=$

k) $1-(-8)=1\quad 8$
$=$

l) $-9-(-2)=-9\quad 2$
$=$

m) $-2-(-3)=-2\quad 3$
$=$

n) $7-(-8)=7\quad 8$
$=$

o) $-6+(-4)=-6\quad 4$
$=$

p) $-6-(-3)=-6\quad 3$
$=$

18) Rewrite and evaluate:

a) $-8-(-7)=$
$=$

b) $10+(-4)=$
$=$

c) $-2+(-6)=$
$=$

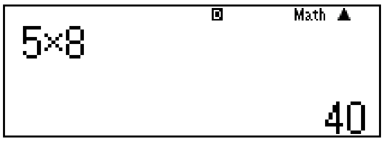
d) $6+(-9)=$
$=$

e) $4-(-9)=$
$=$

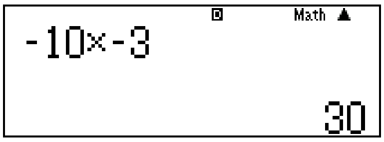
f) $-7-(-4)=$
$=$

19) The rules for multiplying with negatives are easy to learn. Study these four examples:

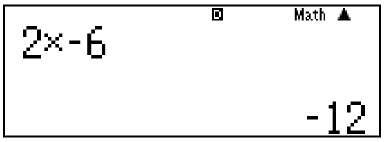
$\underset{\text{pos}}{5}\times\underset{\text{pos}}{8}=40$ (same signs)



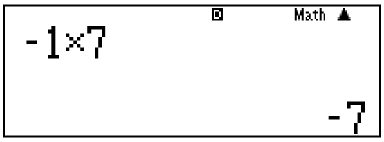
$\underset{\text{neg}}{-10}\times\underset{\text{neg}}{(-3)}=30$ (same signs)



$\underset{\text{pos}}{2}\times\underset{\text{neg}}{(-6)}=-12$ (different signs)



$\underset{\text{neg}}{(-1)}\times\underset{\text{pos}}{7}=-7$ (different signs)



Just look at the signs of the two numbers in the question:

If the numbers have **the same** signs, the answer is positive.
If the numbers have **differen**t signs, the answer is negative.

20a) $4\times7=$

b) $-4\times7=$

c) $4\times(-7)=$

d) $-4\times(-7)=$

e) $-8\times2=$

f) $4\times(-9)=$

g) $1\times(-5)=$

h) $6\times(-6)=$

i) $-2 \times 2 =$

j) $-2 \times (-3) =$

k) $-10 \times 1 =$

l) $11 \times (-2) =$

m) $-5 \times (-4) =$

n) $(-4)^2 = -4 \times -4$

$=$

o) $-5 \times 6 =$

p) $(-1)^2 = \quad \times$

$=$

21) The rules for dividing are exactly the same as for multiplying.
Write down the answers to these divisions.

a) $12 \div 2 =$

b) $12 \div (-2) =$

c) $-12 \div 2 =$

d) $-12 \div (-2) =$

e) $-30 \div (-3) =$

f) $14 \div (-1) =$

g) $36 \div (-6) =$

h) $\frac{50}{-5} =$

i) $\frac{-42}{-7} =$

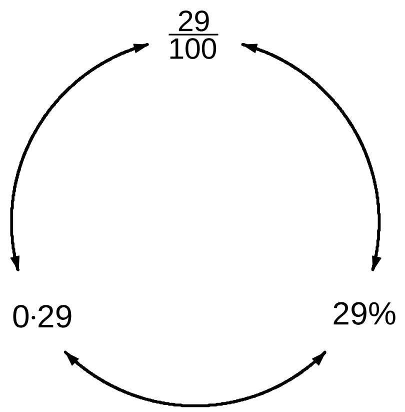
j) $\frac{-6}{-2} =$

k) $\frac{-7}{1} =$

l) $\dfrac{-3 \times (-8)}{-2} =$

Fractions & percentages

22) You should know how to change between fractions, decimals and percentages.



Three different ways of writing the same number. The decimal and the percentage are obviously connected – just $\boxed{\times}$ or $\boxed{\div}$ by 100 depending which one you want.

23) The words 'per cent' literally mean 'out of a hundred'.

This means that a percentage is quite simply a special fraction where the denominator always has a value of 100.

For example $\frac{7}{100}$ is a percentage and can be written as 7%

$$7\% = \frac{7}{100}$$
$$3\% = \frac{3}{100}$$
$$29\% = \frac{29}{100}$$

These particular fractions cannot be simplified or cancelled down.

But here are some that do cancel down:

$$15\% = \frac{15}{100} = \frac{3}{20}$$ (dividing top and bottom by 5)

$$38\% = \frac{38}{100} = \frac{19}{50}$$ (dividing top and bottom by 2)

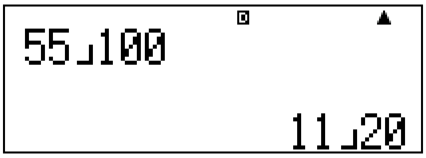
One of the National 5 exam papers is a non-calculator paper, so it is worth knowing how to cancel manually.

24) Cancelling can also be carried out on the calculator by using the $\frac{\square}{\square}$ button.

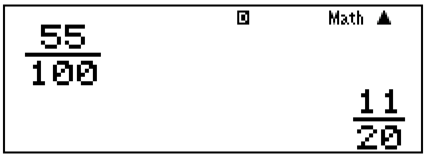
Suppose we want to write 55% in its simplest form. We just need to type in

5 5 $\frac{\square}{\square}$ 1 0 0 =

In Line mode on the Casios it looks like this:



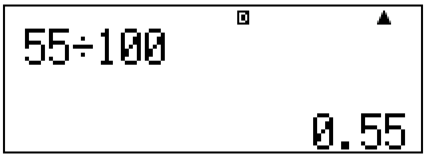
In Math mode it looks more natural:



Either way, 55% simplifies to the fraction $\frac{11}{20}$.

Depending how your calculator is set up, the percentage can also be expressed as a **decimal** by dividing instead

5 5 ÷ 1 0 0 =



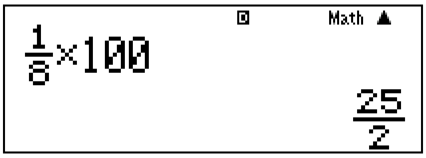
Alternatively toggle between fraction and decimal using the wonderful S⇔D button!

25) We can carry out this operation in reverse and change a fraction into a percentage by multiplying by 100. Look at these examples:

$\frac{1}{2} \xrightarrow{\times 100} 50\%$ $\quad\quad$ $\frac{1}{8} \xrightarrow{\times 100} 12 \cdot 5\%$

If you do this last sum in Math mode, you get

1 $\frac{\square}{\square}$ 8 × 1 0 0 =

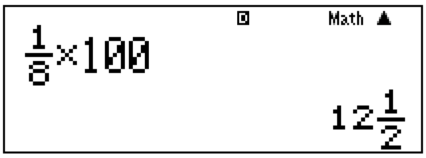


This type of answer is called a top-heavy fraction.
To change it to a “mixed number”, use the

$\left(a\frac{b}{c} \leftrightarrow \frac{d}{c}\right)$

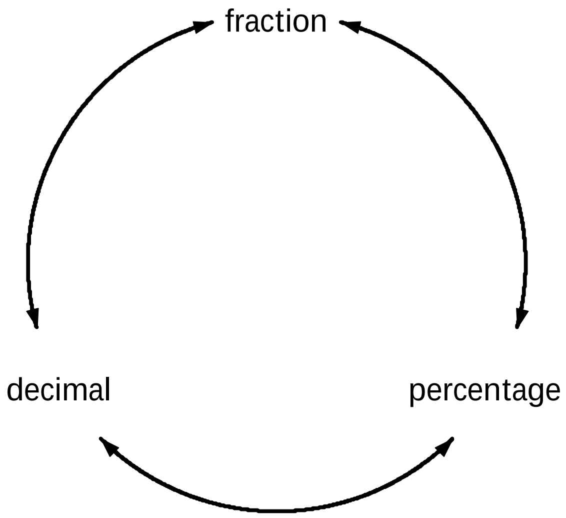
button which is in yellow writing above the S⇔D. Type

SHIFT S⇔D

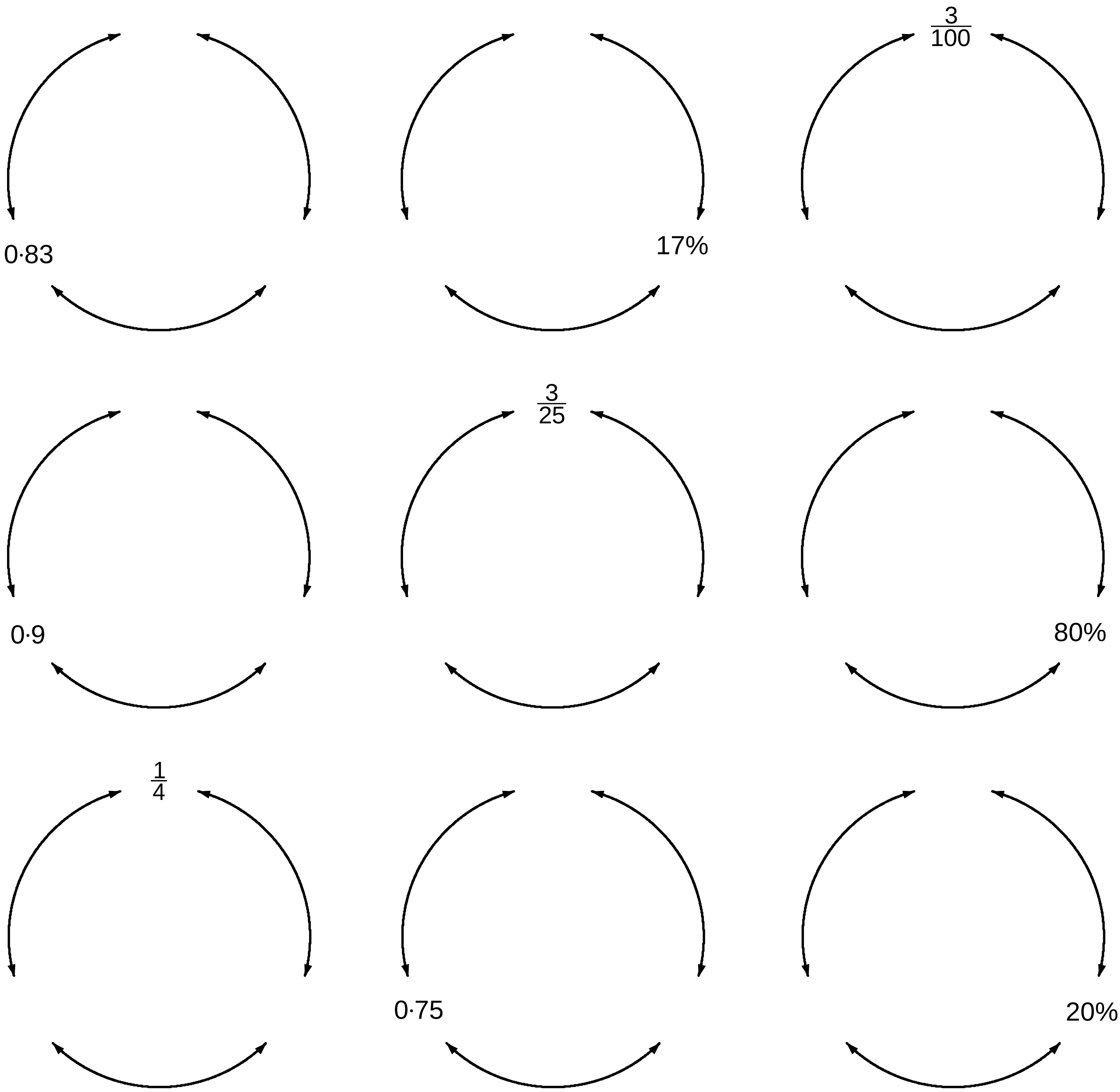


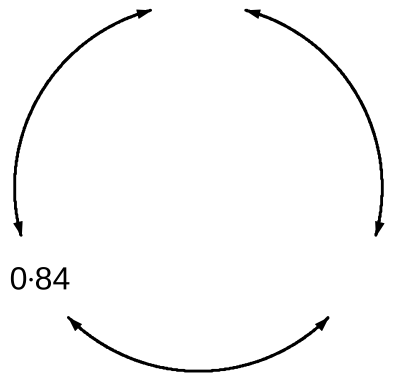
to access it.

26)



Fill in the missing figures. Fractions should be cancelled down when possible.

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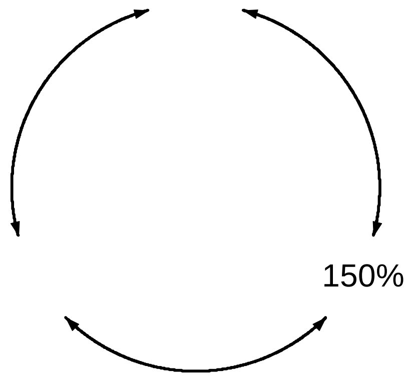
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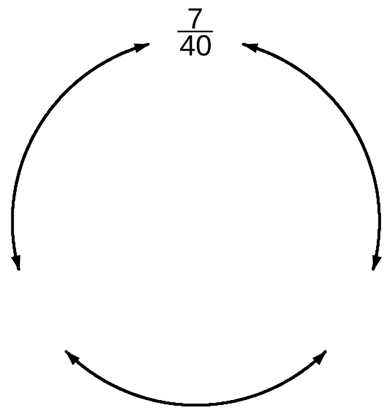
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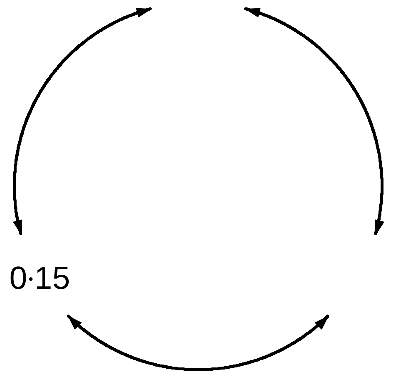
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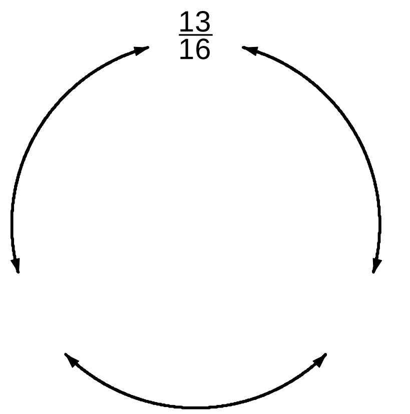
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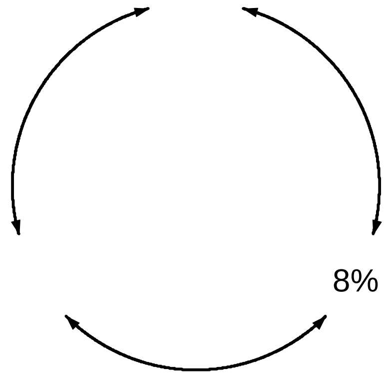
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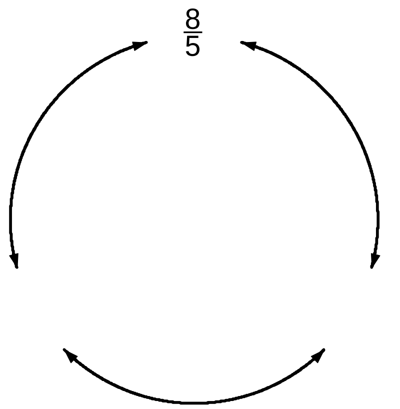
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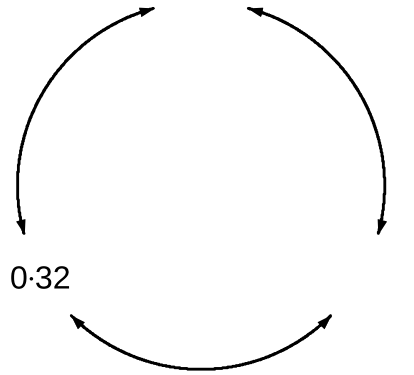
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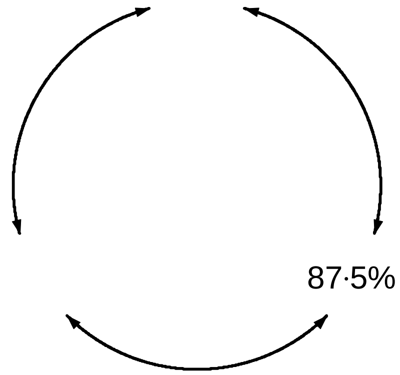
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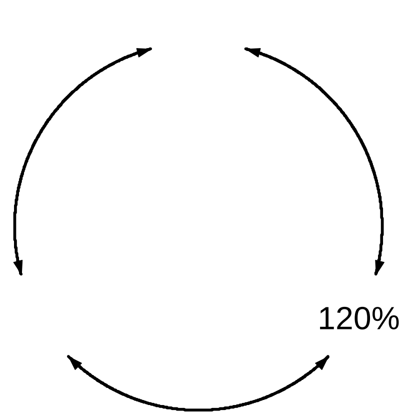
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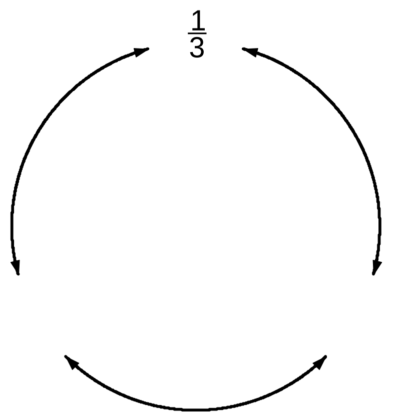
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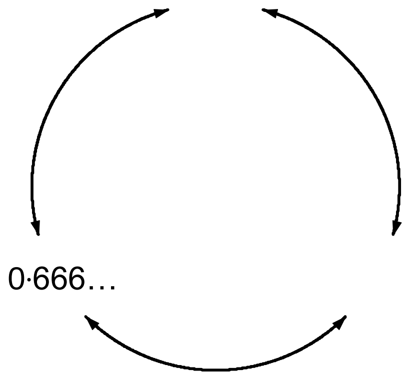

0·666…

27) To find a fraction of a number,

$\boxed{\div}$ by bottom
$\boxed{\times}$ by top

For example,

$$\frac{3}{5} \text{ of } 85 \text{ km} = 85 \div 5 \times 3$$
$$= 51 \text{ km}$$

Percentages are fractions out of 100, so to find a percentage of a number

$\boxed{\div}$ by bottom (100)
$\boxed{\times}$ by top

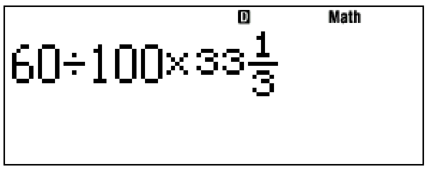
For example,

$$36\% \text{ of } 425 \text{ ml} = \frac{36}{100} \text{ of } 425 \text{ ml}$$
$$= 425 \div 100 \times 36$$
$$= 153 \text{ ml}$$

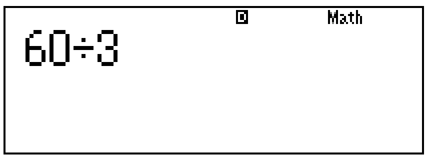
Awkward percentages like $33\frac{1}{3}\%$ can be dealt with in different ways depending on your calculator.

To find $33\frac{1}{3}\%$ of £60 you can choose between using the

button (in yellow above),



or hopefully you already know that $33\frac{1}{3}\%$ means the same as one third so, easiest of all, just do one third of £60.



28) Calculate

a) $\frac{2}{5}$ of $20 = 20 \div \quad \times$
$=$

b) $\frac{3}{4}$ of $1800 = 1800 \div \quad \times$
$=$

c) $\frac{4}{9}$ of $342 =$
$=$

d) $\frac{13}{15}$ of $450 =$
$=$

e) $\frac{5}{6}$ of $120 =$
$=$

f) $\frac{5}{7}$ of $91 =$
$=$

29) Calculate

a) 60% of $35 = \frac{60}{100}$ of 35
$= 35 \div \quad \times$
$=$

b) 12% of $47 \cdot 5 = \frac{\quad}{100}$ of $47 \cdot 5$
$= 47 \cdot 5 \div \quad \times$
$=$

c) 65% of $340 =$

d) $2 \cdot 4\%$ of $25 =$

e) 105% of $300 =$

f) 72% of $385 =$

g) (see note on previous page)
$33\frac{1}{3}\%$ of $48 =$

h) $66\frac{2}{3}\%$ of $1485 =$

30a) There are 85 cars in the college car park. $\frac{3}{5}$ of them belong to staff. How many is this?

b) Harry weighs 75 kg and goes on a diet. He loses 8% of his weight
How many kg did Harry lose?

c) Inez earned £8·25 an hour. She got a 4% pay rise.
What was her new hourly wage?

Algebra shorthands

31) There are two important shorthands to know.

First, when a letter sits next to a number like $3r$, or a letter sits next to another letter like aw, this means there is a missing “×” sign.

A times sign is the **only** symbol you are allowed to miss out in maths.
$3r$ means 3 times r or 3 lots of r. We can write $3r$ as $3\times r$ or $r+r+r$ if we want.

We always write the number first, then the letter.
If there is more than one letter, they usually go in alphabetical order.

$4e = 4\times e$ or $e+e+e+e$

$12n = 12\times n$ or $n+n+n+n+n+n+n+n+n+n+n+n$

$pw = p\times w$ $\quad$ $abc = a\times b\times c$ $\quad$ $5dk = 5\times d\times k$

Secondly, if we multiply a letter by one or more copies of itself we use power notation.

$b\times b = b^2$ $\quad$ $v\times v = v^2$ $\quad$ $s\times s\times s = s^3$

This brings us to commonly misunderstood shorthands like $3d^2$.

$3d^2 = 3\times d\times d$ $\quad$ $\pi r^2 = \pi\times r\times r$ (you will see this formula later)

Note that $3d\times 3d$ could be written as $(3d)^2$ but not $3d^2$.

32) Write the correct mathematical shorthand for these:

a) $8\times m =$

b) $15\times q =$

c) $g\times 4 =$

d) $k+k+k+k+k =$

e) $c\times d =$

f) $z\times a =$

g) $2\times f\times g =$

h) $t\times 8\times s =$

i) $u\times u =$

j) $a\times a\times a\times a =$

k) $6\times h\times h =$

l) $7e\times 7e =$

Using a formula

33) A formula is a rule that always works. The rule is written in algebra shorthand, and you are usually given numbers to replace the letters with. The final answer will be a number.

It is easy to do formulas, even complicated ones, on the Casios, if you know your buttons. Here are some examples, please fill in the final answers:

a) $F = 4r + j$ is a formula. Find the value of F when $r = 12$ and $j = 3$.

Wrap each number you substitute in brackets:

$$
\begin{aligned}
F &= 4r + j \\
&= 4(12) + (3) \\
&=
\end{aligned}
$$

D Math ▲

$$4(12)+3$$

b) $y = 5 - 6x - x^2$

Find the value of y when $x = -2$.

Ones like this are a bit harder to get correct without a calculator.

$$
\begin{aligned}
y &= 5 - 6(-2) - (-2)^2 \\
&=
\end{aligned}
$$

D Math ▲

$$5-6(-2)-(-2)^2$$

c) $V = \frac{4}{3}\pi r^3$

Find the value of V when $r = 5$.

For maximum accuracy with minimum fuss, use your π button in formulas like this.

$$
\begin{aligned}
V &= \tfrac{4}{3}\pi r^3 \\
&= \tfrac{4}{3}\pi(5)^3 \\
&= \qquad\qquad \text{to 2 dp}
\end{aligned}
$$

D Math ▲

$$\frac{4}{3}\pi(5)^3$$

d) $h = \dfrac{4a - a^3}{1 - 2a^2}$

Find the value of h when $a = \frac{1}{2}$.

$$
\begin{aligned}
h &= \frac{4a - a^3}{1 - 2a^2} \\
&= \frac{4\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^3}{1 - 2\left(\frac{1}{2}\right)^2} \\
&=
\end{aligned}
$$

D Math ▲

$$\frac{4\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)^3}{1-2\left(\frac{1}{2}\right)^2}$$

34a) $y = 6v^2$

Calculate the value of y when $v = 3$

b) $m = \frac{5}{6}k^2$

Calculate the value of m when $k = 12$

c) $T = \frac{3}{10}(\pi + 2a)$

Calculate the value of T when $a = 8$

d) $g = 3\sqrt{w}$

Calculate the value of g when $w = 25$

e) $Q = \sqrt{30 - 4f}$

Calculate the value of Q when $f = 2 \cdot 5$

f) $N = \sqrt{\dfrac{r + 7}{2c}}$

Calculate the value of N when $r = 29$ and $c = 2$

g) $E = \frac{1}{3}bd^2$

Calculate the value of E when $b = 24$ and $d = 1 \cdot 5$

h) $h = \frac{4\pi}{\sqrt{s}}$

Calculate the value of h when $s = 10$

i) $j = \frac{2}{7}\left(5x + \sqrt{y}\right)$

Calculate the value of j when $x = 2 \cdot 1$ and $y = 196$

j) $D = \frac{1}{8\pi mn}$

Calculate the value of D when $m = 2$ and $n = 0 \cdot 05$

k) $p = \sqrt{6d + \frac{t^2}{u - 1}}$

Calculate the value of p when $d = 9$, $t = 80$ and $u = 5$

Area of a triangle

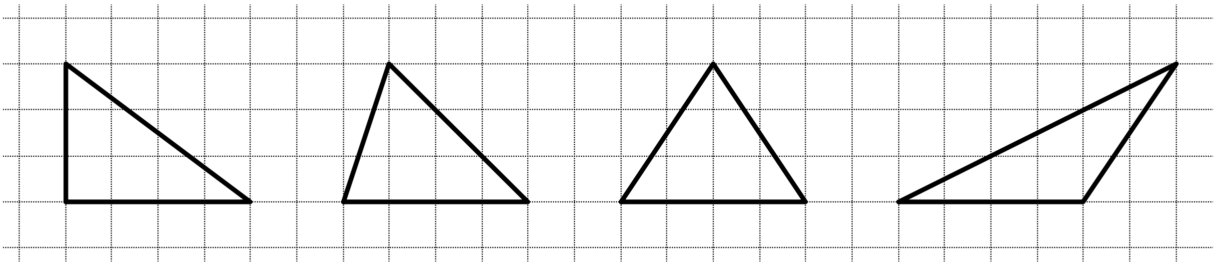
35) The area of a triangle is given by the formula

$$A = \tfrac{1}{2}bh$$

$\frac{1}{2}$ of base $\times$ height.

b means the whole base, *h* means the height at right-angles to the base.

36) Here are four triangles which all have **exactly the same base and height**.

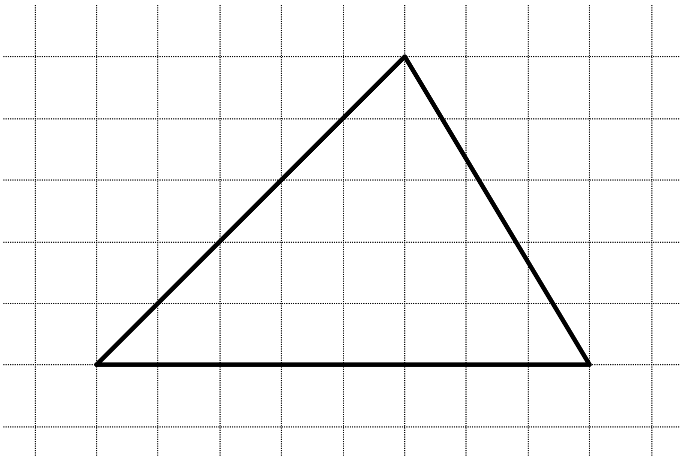


a) What is the base of each triangle? (Count squares, don't measure with a ruler.)

b) What is the height of each triangle?

c) Work out the area of each triangle.

37a) How many squares long is the base of this triangle?

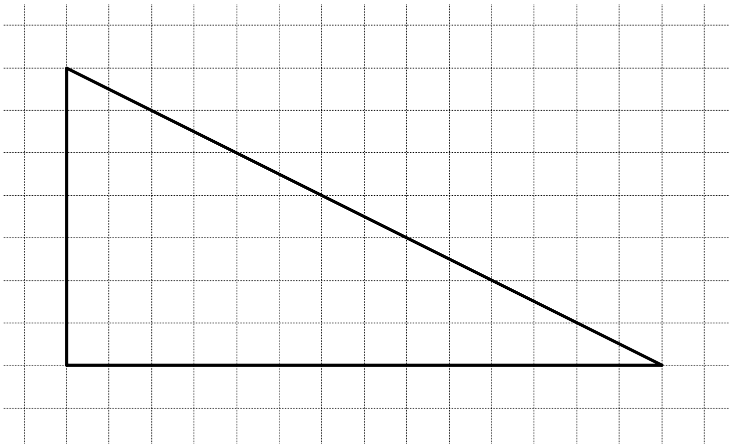


b) How high is this triangle?

c) Work out the area of this triangle:

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \text{ of } \qquad \times \\ &= \frac{1}{2} \text{ of } \\ &= \end{aligned}$$

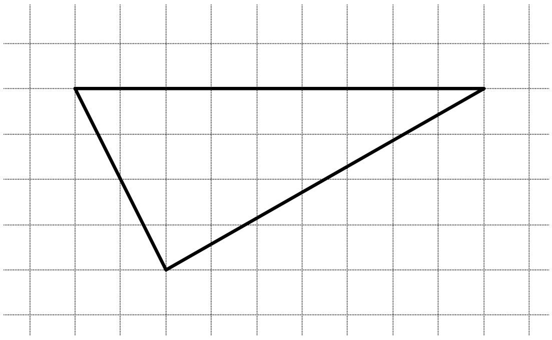
38a) How long is the base of this triangle?



b) How high is this triangle?

c) Show your own working to calculate the area:

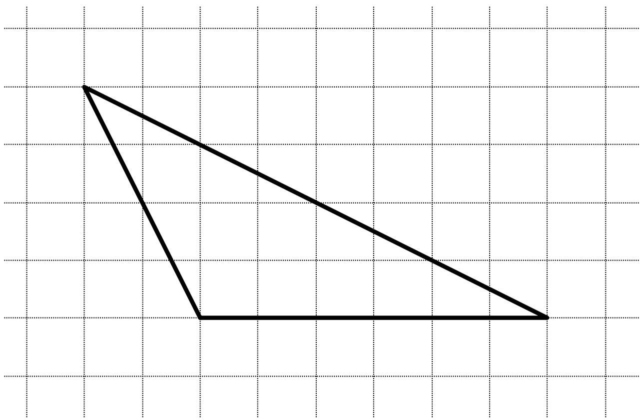
39) How long is the base of this triangle?



b) How high is this triangle?

c) Show your own working to calculate the area:

40a) How long is the base of this triangle?

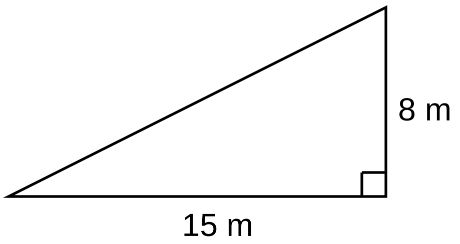


b) How high is this triangle?

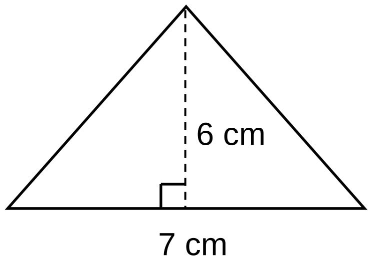
c) Show your own working to calculate the area:

41) Calculate the area of each triangle. The answers will be in square metres (m^2) and so on.

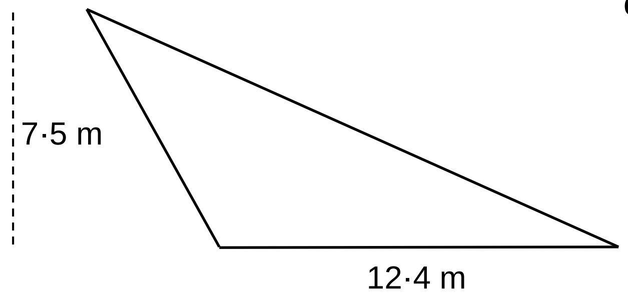
a)



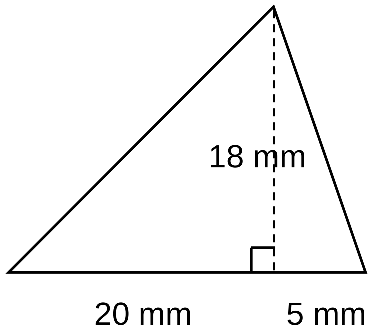
b)



c)



d) (Think of a quick way to do this one)



42) If you have to do $\frac{1}{2}bh$ without a calculator, look for an even number you can easily take half of first.

For example, if you have to do $\frac{1}{2}$ of 13×20, begin with $\frac{1}{2}$ of 20 and then multiply that by 13.

Work these areas out **without** a calculator:

a) $\frac{1}{2}$ of 13×20

b) $\frac{1}{2}$ of 26×5

c) $\frac{1}{2}$ of 18×5

d) $\frac{1}{2}$ of 25×8

e) $\frac{1}{2}$ of 33×6

f) $\frac{1}{2}$ of 11×12

Simplifying

43) The main idea here is what we call “counting like terms”.

If you have 3*c*, that means 3 lots of the thing called *c*. If you then introduce an extra 5*c*, you now have 8 things called *c*, in other words 8*c*, altogether.

$$3c + 5c = 8c$$

The two terms on the left-hand side are called **like terms**, though "alike" terms might be a better phrase. They both feature some amount of *c*'s.

However, a mixture like

$$3c + 5d$$

cannot be simplified because these are not like terms. If you had 3 cats and 5 dogs, you cannot say you have 8 "catdogs" or anything like that.

Like terms can be added and subtracted from each other. You are just counting them up.

Here are some easy examples to study:

- $2a + 3a = 5a$
- $9m + 5n$ cannot be simplified.
- $7x - 5x = 2x$ Here you are **reducing** the number of *x*'s by 5.
- $6m + m = 7m$ If you see plain "*m*", it means **one** of them, 1*m*.
- $10f - 9f = f$ Write "*f*" instead of 1*f*.
- $8r - 8r = 0$ We have no *r*'s left. It would be inefficient to write 0*r*.
- $4u - 5u = -u$ Again “$-1u$” is sloppy shorthand, write "$-u$" instead.
- $6e - 9e = -3e$ We are removing more *e*'s than we started with, so we go into the negatives.
- $-4p + 2p = -2p$ You could check by typing $\boxed{(-)}\ \boxed{4}\ \boxed{+}\ \boxed{2}\ \boxed{=}$
- $-3y - 4y = -7y$ You could check by typing $\boxed{(-)}\ \boxed{3}\ \boxed{-}\ \boxed{4}\ \boxed{=}$

44) Simplify the following expressions where possible.

a) $3a+2a=$

b) $3x-14x=$

c) $y+7y=$

d) $12a-8c=$

45) If you saw 3 cats and 6 dogs on Saturday, then 4 cats and 7 dogs on Sunday, you would have seen ____ cats and ____ dogs that weekend.

As an algebra equivalent, we could write

$$3c+6d+4c+7d=7c+13d\,.$$

It's as simple as counting the different animals separately. You didn't see 20 "catdogs", so we can't call this $20cd$ or anything daft like that.

46) Note the rules of negatives sometimes come into play, and you may find it helpful to rearrange the terms. For example, simplify

$$a-2b+4a-9b-2a$$

we must count the *a*'s then separately count the *b*'s. Be careful, the sign before a term "belongs" to it and, if you change the order the sign goes with the term.

$$\begin{aligned} a-2b+4a-9b-2a &= \underbrace{a+4a-2a}_{\text{count the } a\text{'s}}\underbrace{-2b-9b}_{\text{count the } b\text{'s}} \\ &= 3a-11b \end{aligned}$$

47) Simplify $-5p+4u+2p-10u$.

Without changing the order, for the number of *p*'s we must work out $-5+2$; for the *u*'s we evaluate $4+(-10)$.

$$-5p+4u+2p-10u=-3p-6u$$

48) Simplify

a) $7x + 10y + 2x + 3y =$

b) $5e + 2k + 2e + 2k =$

c) $9m + 3r + 3m - 8r =$

d) $2h + 8b - 6h - 3b =$

e) $-12u - 7v + u + 5u - v =$

f) $-2p + q + 8p - 4q =$

g) $-9n - 6j + 11n + 5j =$

h) $3a - 2b + 4b - 2a + 3b - 2a - 3b =$

49) If you see a plain number somewhere in a lump of algebra, such as “5”, that just means 5 “units”. Count up units separately from any letters. For example,

$$4h + 5 + 8h + 2 = 12h + 7$$

50) Simplify

a) $3k + 8 + k + 2 =$

b) $6d + 1 - 5d + 3 =$

c) $9 + 3g - g - 12 =$

d) $-12f + 4 + 6f + 3 =$

e) $2b + 2 - 9 - b =$

f) $5s - 3 - 5s - 3 =$

51) It is common to get confused when the terms themselves have more complicated names. For example, how do you simplify $2h^2 + 5h^2$?

Maybe $7h^4$?

Afraid not. If you have two things called "h^2" and then five more things called "h^2", you just have seven things called h^2.

$$2h^2 + 5h^2 = 7h^2$$

A more extreme example would be

$$abc + 5\sin^2\theta - 7abc + 3\sin^2\theta$$

It doesn't matter how complicated the name of an individual term is, you just need to ask if there are any more of them to count up.

$$abc + 5\sin^2\theta - 7abc + 3\sin^2\theta = -6abc + 8\sin^2\theta$$

As you know, some things cannot be simplified. For example $6ut - 3t^2$, because a "*ut*" is a different creature to a t^2.

52) Simplify

a) $4k^3 + 5k^3 =$

b) $y^2 + 3y + 2 + 2y^2 - y + 11 =$

c) $5fg - r^2 - fg - r^2 =$

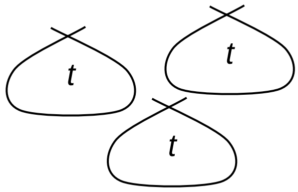
d) $v^2 - 8 + 3v^2 - v + 10 =$

e) $3\sin\alpha\cos\psi - 6\sin\alpha\cos\psi =$

f) $2\pi x^2 + 2\pi xh - \pi x^2 =$

Brackets

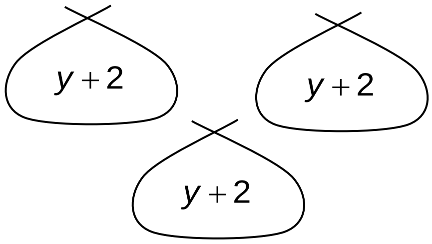
53) If we stick a “3” in front of a letter, like $3t$, then this means three **lots of** *t*. We multiply. Here is a picture representing what $3t$ means:



If we stick a “3” in front of a more complicated lump of algebra, this means three lots of that lump of algebra. The only condition is we have to write the lump of algebra inside a bracket.

For example, $3(y+2)$ means 3 lots of the lump of algebra $(y+2)$. The brackets here have been represented by “bags”:

$3(y+2) =$



If you count up everything in the picture, you can see there are three *y*’s and also three 2’s. So

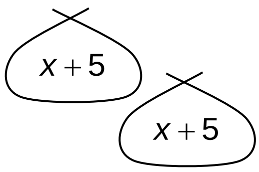
$$3(y+2) = 3y+6$$

When we rewrite $3(y+2)$ as $3y+6$, it is called multiplying out or **expanding** the brackets.

54) Here are some more examples of expanding brackets : the arrows show us to multiply every term inside the bracket by the number outside.

$2(x+5)$

$= 2x+10$



$6(y-1)$

$= 6y-6$

$3(2x+5)$

$= 6x+15$

$7(2r+5)$

$= 14r+35$

55) Expand out:

a) $2(x+3)=$

b) $3(r+6)=$

c) $5(n+1)=$

d) $10(v-4)=$

e) $8(k-2)=$

f) $7(g-1)=$

g) $4(2b+2)=$

h) $2(6c+3)=$

i) $2(7h-3)=$

j) $8(3e+5f)=$

56) There may be more than two terms inside the bracket:

$$2(10p-7q+11r)$$

$$=20p-14q+22r$$

Expand out:

a) $4(f+3g+5)=$

b) $5(3a+b-2c)=$

57) Sometimes you might be asked to multiply a bracket by a **negative** number.
Take a moment to read back over the rules from q19).

When you multiply by a negative, basically all the terms from the bracket **change their sign** as well as getting bigger.

$$-3(2x+7)$$
$$=-6x-21$$

$$-5(a-4)$$
$$=-5a+20$$

In the second example, the 20 is positive because when we multiply -5 by -4, both signs are the same.

58) Expand out:

a) $-5(n+2)=$

b) $-8(f-2)=$

c) $-3(-b+3)=$

d) $-2(v+1)=$

e) $-6(2k-5)=$

f) $-9(-4d+e)=$

g) $-(c+3)=$
[this means $-1(c+3)$]

h) $-(-1-g)=$

59) Study these which have letters outside the brackets as well.
The rules are just the same – multiply all terms inside by the letter outside.
The middle line of working is optional. Don't forget the "squaring" shorthand!

$x(y+2)$	$2x(x+3)$	$4d(d-3e)$
$= x \times y + x \times 2$	$= 2x \times x + 2x \times 3$	$= 4d \times d - 4d \times 3e$
$= xy + 2x$	$= 2x^2 + 6x$	$= 4d^2 - 12de$

60) Expand:

a) $a(b+4) =$

b) $m(n+p) =$

c) $3x(x+2) =$

d) $4r(r-2) =$

e) $3x(2x+1) =$

f) $-p(-q+2t) =$

g) $-s(5-2s) =$

h) $-2k(2k-3) =$

Removing brackets and collecting like terms

61) Now we take things further.

If a term is **not** inside the bracket you don't multiply it! After you have expanded the bracket, tidy up the list of terms like you did in q48) and 50).

For example, simplify $2(a+3)+1$

$$\begin{aligned} &2(a+3)+1 \\ &= 2a+6+1 \\ &= 2a+7 \end{aligned}$$

The "1" is not in the bracket so must not multiply it by 2.

Study these examples:

$$\begin{aligned} &4(b+5)-19 \\ &= 4b+20-19 \\ &= 4b+1 \end{aligned}$$

$$\begin{aligned} &6(x+2)-4x \\ &= 6x+12-4x \\ &= 2x+12 \end{aligned}$$

$$\begin{aligned} &3v+2(6v-2s) \\ &= 3v+12v-4s \\ &= 15v-4s \end{aligned}$$

This one has "more" in it but is probably easier:

$$\begin{aligned} &2(y+1)+4(5y+2) \\ &= 2y+2+20y+8 \\ &= 22y+10 \end{aligned}$$

62) Multiply out and simplify:

a) $8(t+2)-6=$

$=$

b) $3(2c-1)-6c=$

$=$

c) $4(2m+3)+3=$

$=$

d) $5x+3(x+4)=$

$=$

e) $3r+2(r-1)=$

$=$

f) $8t+2(2t-9)=$

$=$

g) $5(3d+2)+d-8=$

$=$

h) $-2(w-9)+3w=$

$=$

63) Be careful with ones like this:

$$3+4(a+2)$$

Look how it begins – we all know $3+4=7$ but that's not what is meant here.
It's not 7 lots of the bracket; it's 3 units then 4 lots of the bracket. Due to BODMAS (don't worry if you don't know what that means), we must multiply the bracket by 4 first.

$$\begin{aligned} &3+4(a+2) \\ &=3+4a+8 \\ &= \end{aligned}$$

64) Simplify

a) $2+5(2p+1)=$

$=$

b) $4+4(2h+3)+9h=$

$=$

Common factors

65) The last section involved multiplying out brackets. Now we're going to reverse the process and put algebraic expressions into brackets. We call this process factorising.

Remember this?

$$2(x+3)=2x+6$$

The reverse process is

$$2x+6=2(x+3)$$

The “2” before the bracket is called the common factor. It is the largest number that divides into both 2 and 6.

You then ask yourself

"What do I multiply 2 by to get 2*x*?"

"What do I multiply 2 by to get 6?"

The answers to these questions are *x* and 3, so these go in the second bracket.

66) Look at these examples:

$$3x-9=3(x-3)$$

$$4g+12=4(g+3)$$

You can always check the answer by multiplying the brackets out to see if you get back to the original expression.

67) Factorise the following. The first few give you a pointer.

a) $3x+12=3(\quad+\quad)$

b) $5a+5=5(\quad+\quad)$

c) $10y+15=5(\quad+\quad)$

d) $ab+ac=a(\quad+\quad)$

e) $18+8m=2(\qquad)$

f) $2t-4=\quad(\quad-\quad)$

g) $2y+6=$

h) $7w-7=$

i) $5a+15=$

j) $6k+8=$

k) $9i-12=$

l) $9h-6=$

m) $22j+33=$

n) $12r-3=$

o) $14+21g=$

p) $pr+pu=$

q) $ax-bx=$

r) $cd-de=$

s) $12u+9=$

t) $9d-21=$

u) $4x+6=$

v) $pqr+2pq=$

68) There is one thing to watch for in these kind of questions. Suppose we are asked to factorise

$$6n + 30$$

There are several possible answers:

$$6n + 30 = 2(3n + 15)$$

$$6n + 30 = 3(2n + 10)$$

$$6n + 30 = 6(n + 5)$$

but one is better than the others. It's the last one. In fact the other answers would be marked as wrong even though "technically" they are correct.

Always make sure the common factor in front of the bracket is **as big as possible**.

69) Factorise the following, making sure to find the highest common factor.

a) $20f + 30 =$

b) $12m + 18 =$

c) $8s + 16 =$

d) $60h - 80 =$

e) $12j - 16 =$

f) $250y - 100 =$

g) $45p - 18 =$

h) $24g + 120 =$

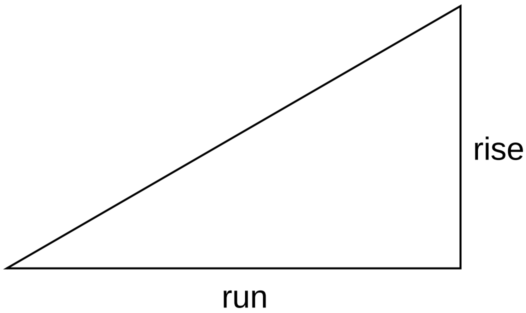
Gradient

70) The gradient of a slope is a measure of its steepness. The gradient is just a number.

It is worked out by dividing the vertical “rise” by the horizontal “run”.

For some reason we use the letter “*m*” to stand for the word gradient:

$$m = \frac{\text{vertical height}}{\text{horizontal distance}} = \frac{\text{rise}}{\text{run}}$$

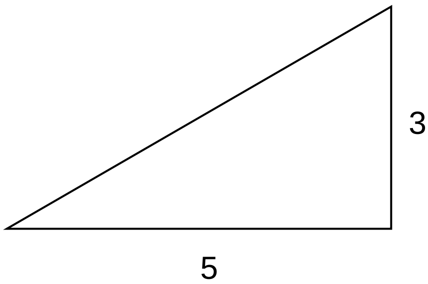


The gradient is normally left as a fraction such as $\frac{3}{4}$, but it can also be turned into a decimal like 0·75.

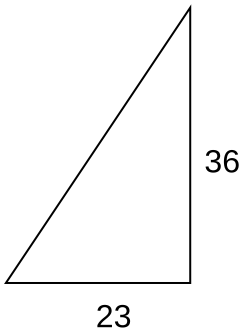
If you are answering as a fraction, always cancel it down where possible. See q24).
If your fraction is something like $\frac{10}{2}$, it cancels down to a whole number 5.

71) Write down the gradients of the following slopes. Leave your answers as fractions in their simplest form where possible.

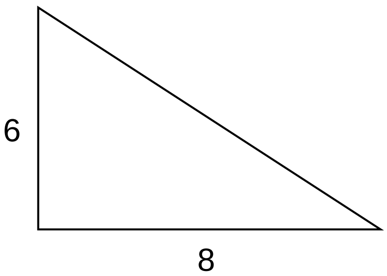
a)



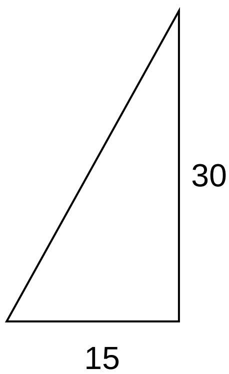
b)



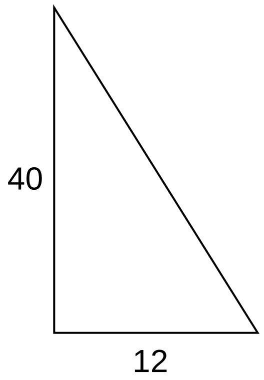
c)



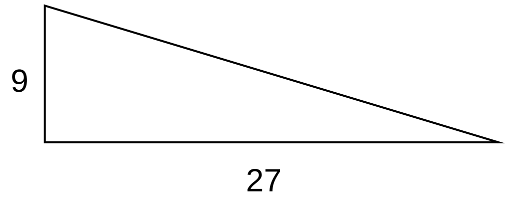
d)



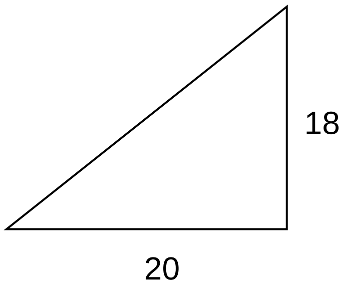
e)



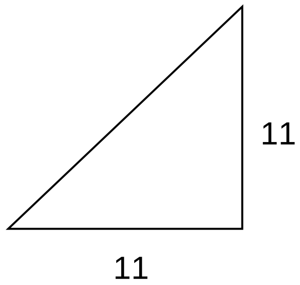
f)



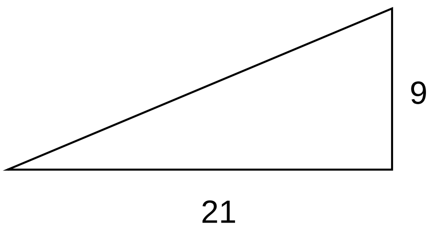
g)



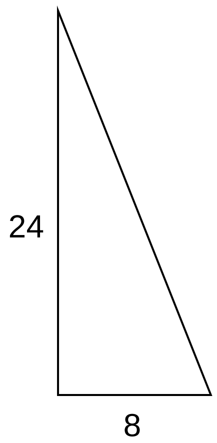
h)



i)

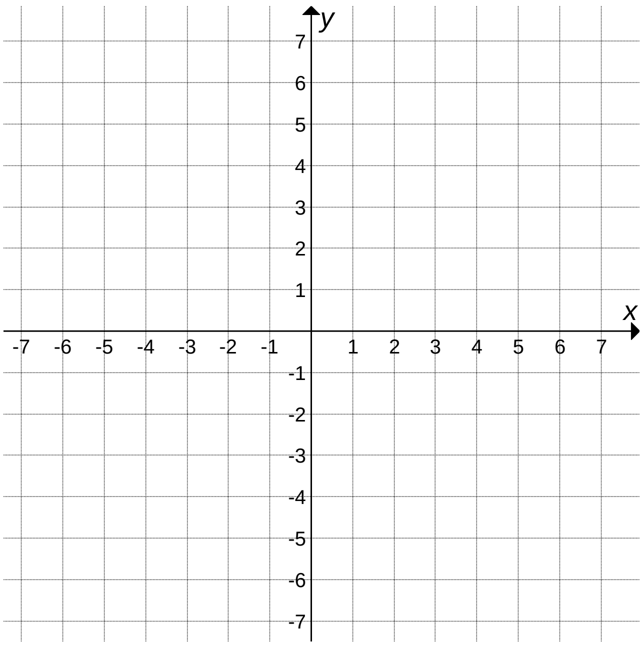


j)

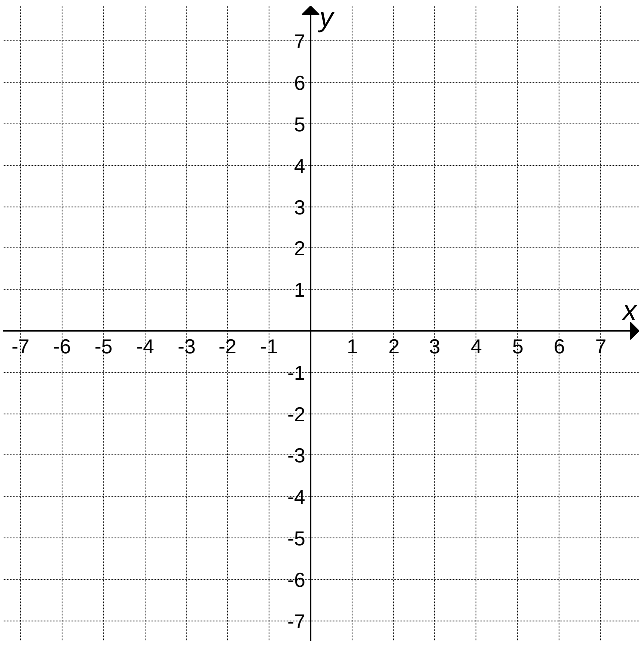


72) Plot each pair of points and calculate the gradient of the line between them in simplest form.

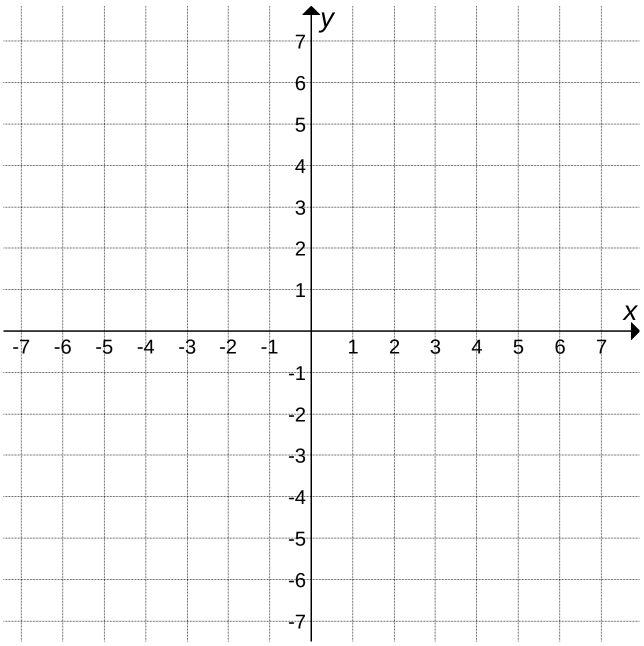
a) A(2, 1) and B(7, 4)



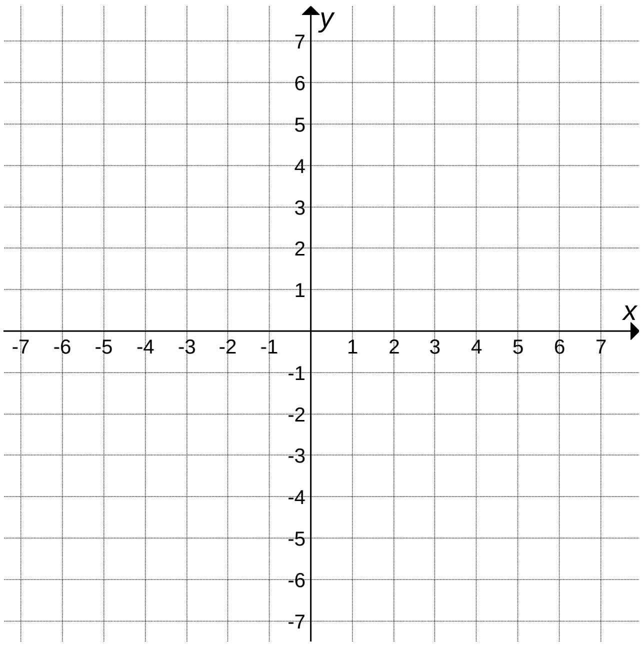
b) M(1, −5) and N(5, 3)



c) G(−6, −6) and H(0, 3)



d) M(−5, 0) and N(7, 2)



Equation of a line

73) The equation of a line is like its DNA : it tells you everything you need to know about the line and allows you to draw it on a graph if you want.

The simplest way to plot a line given its equation is to do some arithmetic. Choose at least three *x*-values and work out the corresponding *y*-values. Then plot each pair as a coordinate and join your points up.

For example to draw the line $y = x + 2$, you might choose 0, 1 and 2 as your *x*-values:

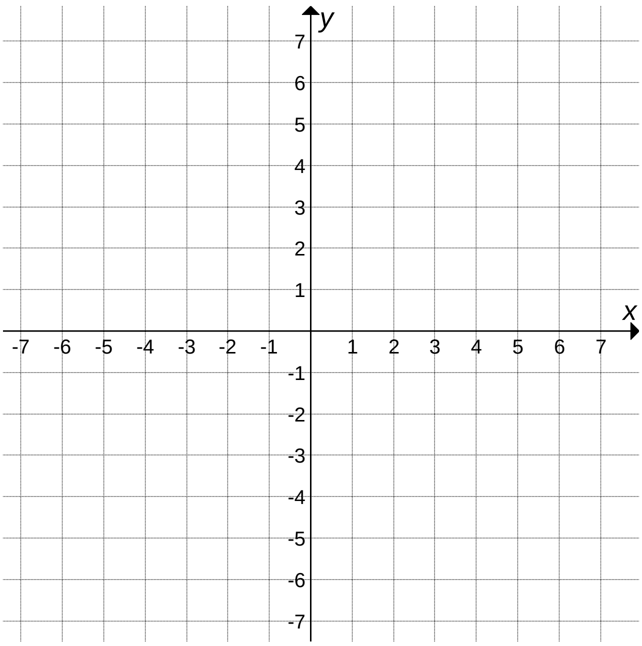
x	0	1	2
y			

There is nothing to stop you choosing other numbers as long as the answers will fit on your graph.

Here are the answers with the working (not normally needed):

x	0	1	2
y	$y = (0) + 2$ $= 2$	$y = (1) + 2$ $= 3$	$y = (2) + 2$ $= 4$

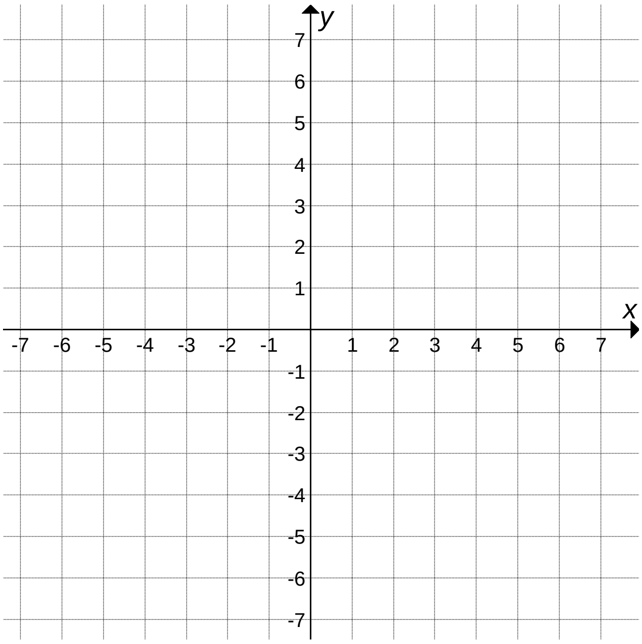
74) Now plot (0, 2), (1, 3) and (2, 4).



Join them up and extend your line both ways to the edges of the grid.
Label your line with its equation.

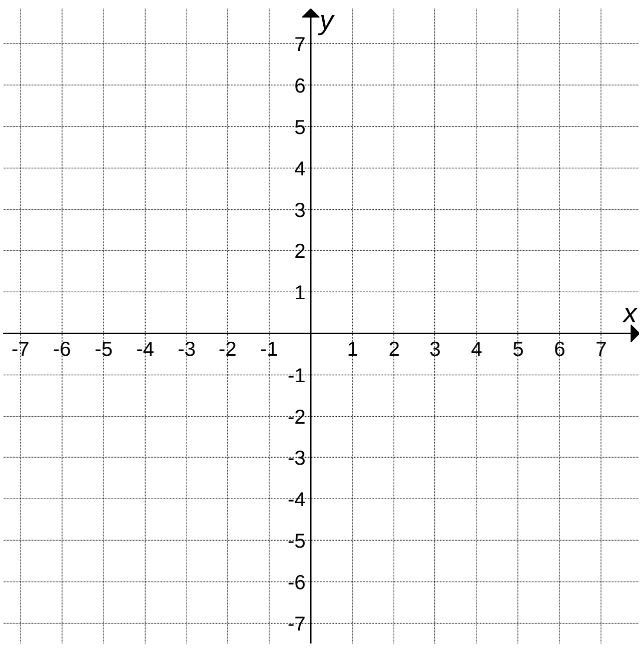
75) Complete the working and draw the line $y = 2x - 3$.

x	0	1	2
y	$y = 2(0) - 3$ $= -3$	$y = 2(1) - 3$ $=$	$y = 2(2) - 3$ $=$



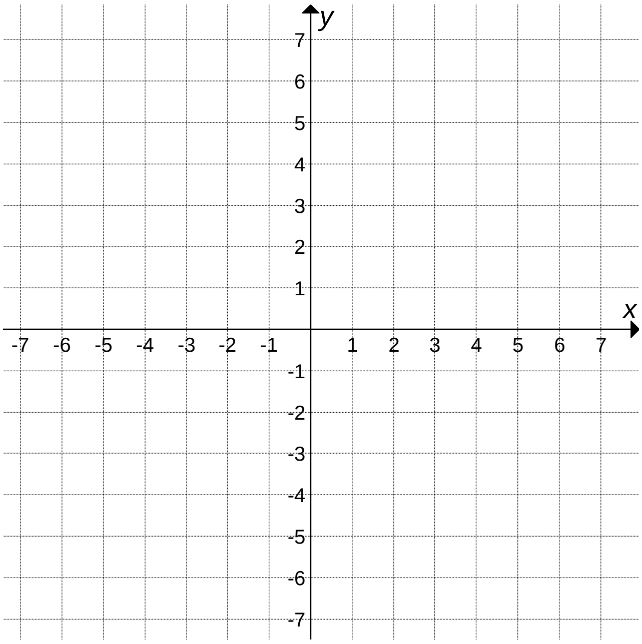
76) Choose your own numbers to draw the line $y = 3x + 1$.

x			
y			



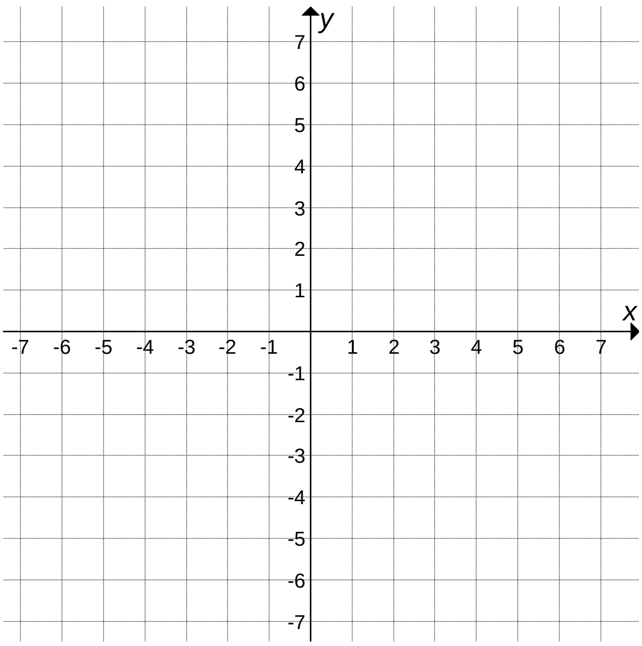
77) If you are asked to draw a line like $y = \frac{1}{2}x + 3$, choose numbers you can take half of!

x	-2	0	4
y	$y = \frac{1}{2}(-2) + 3$ $=$	$y = \frac{1}{2}(0) + 3$ $=$	$y = \frac{1}{2}(4) + 3$ $=$



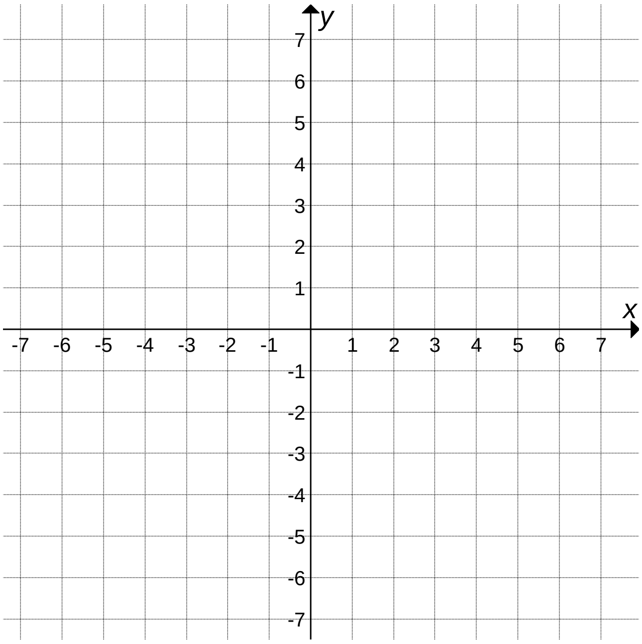
78) Take care when multiplying by a negative. See q19) and 20).
Choose your own numbers and draw the line $y = -2x + 4$.

x			
y			

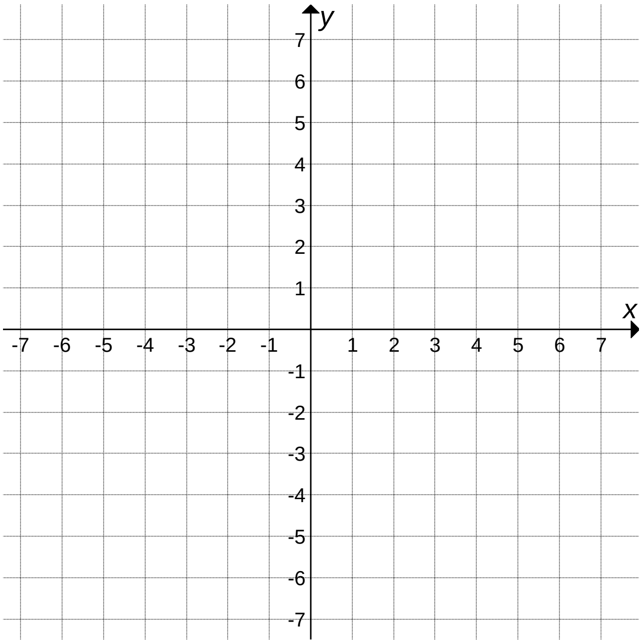


79) Make your own tables and draw the following lines.

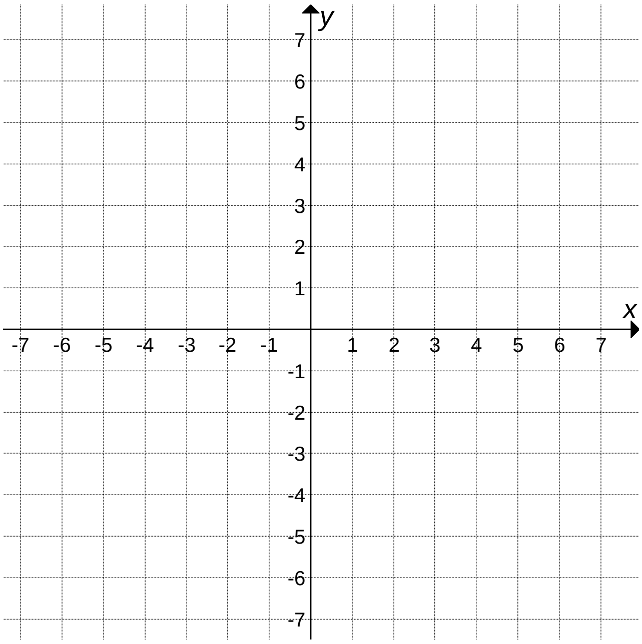
a) $y = x - 3$



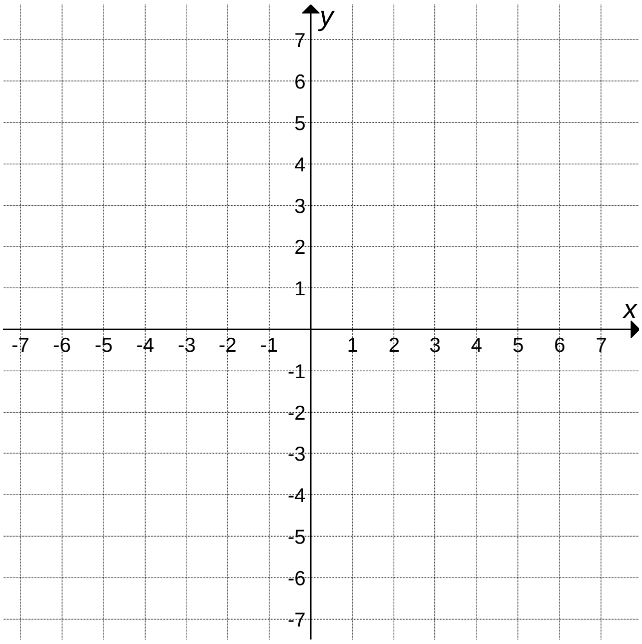
b) $y = \frac{2}{3}x + 2$ (pick nos divisible by 3)



c) $y = -x + 4$

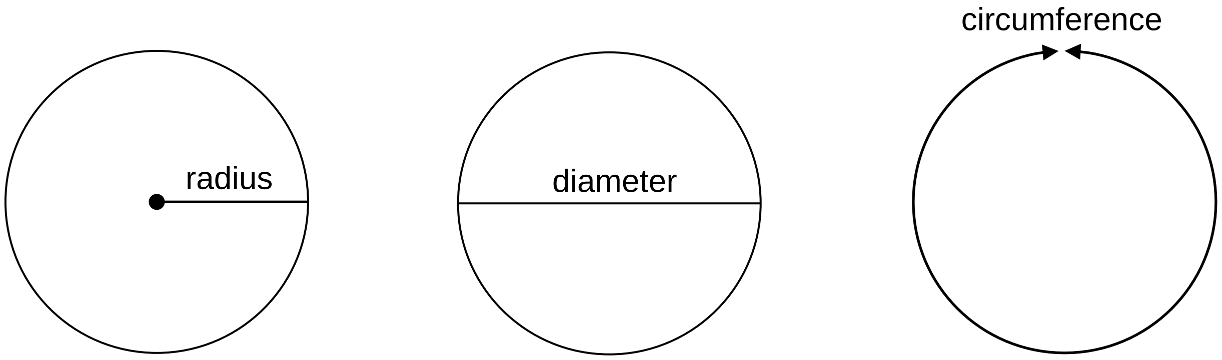


d) $y = -2x - 5$



Circumference and area of a circle

80) Make sure you know these three words of vocabulary:

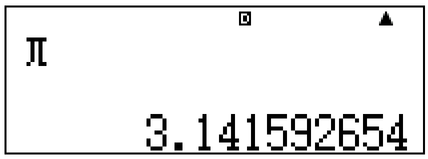


Obviously the diameter of a circle is always twice the radius. However the main idea is that the curved circumference is always just over three times the length of the diameter.

This number, “just over three”, is so special it gets its own name and symbol:

π (“pi”)

π is roughly 3·14. If you press the button marked (π) on your calculator, [SHIFT] [×10^x] [=], a fuller value is displayed.



We can calculate the circumference using one of these formulas:

$$\boxed{c = \pi d}$$

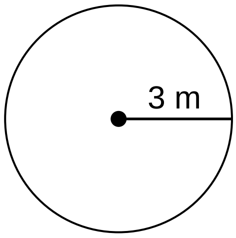
or

$$\boxed{c = 2\pi r}$$

In a circle question we are either told the diameter or the radius, and if you know one you know the other.

It is easiest to use the (π) button on your calculator to carry out any circle calculations, but if you use 3·14 or even $\frac{22}{7}$ you will still be marked correct.

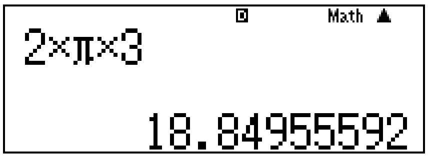
81) Suppose the radius of a circle in a sports arena is 3 metres.
How can we calculate its circumference?

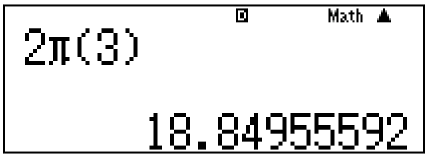


Since we are told the radius,

$$\begin{aligned} c &= 2\pi r \\ &= 2 \times \pi \times 3 \\ &= 18 \cdot 85 \text{ m to 2dp} \end{aligned}$$

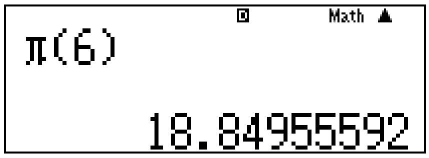
Typing the sum into a calculator is very easy compared to some of the formulas you used in q33) and 34). Here are a couple of different ways you could do it:





Of course you can use the other formula as long as you realise the diameter is 6 metres:

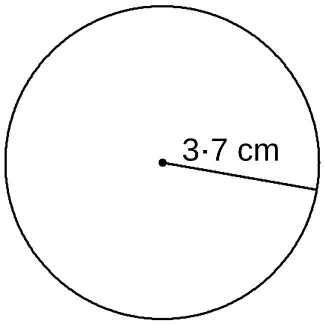
$$\begin{aligned} c &= \pi d \\ &= \pi \times 6 \\ &= 18 \cdot 85 \text{ m to 2dp} \end{aligned}$$



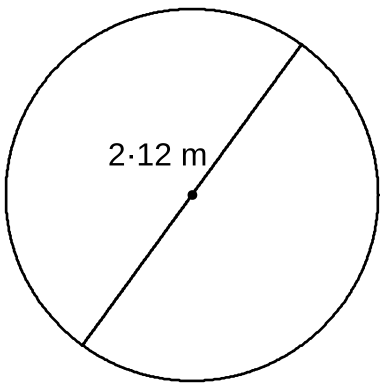
Always round the final answer off sensibly.

82) Calculate the circumference of each circle.

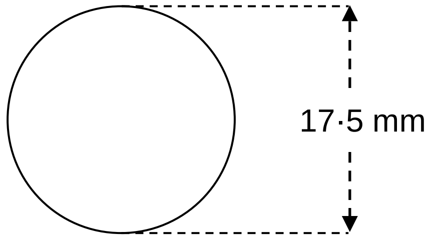
a)



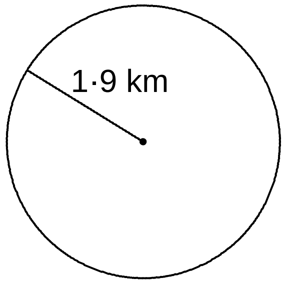
b)



c)



d)

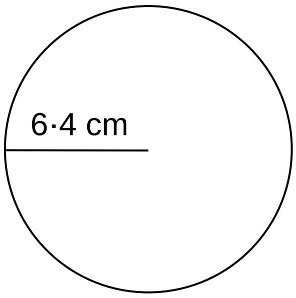


83) To calculate the area of a circle we have to use a new formula:

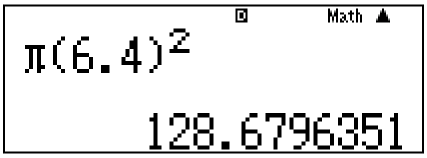
$$\boxed{A = \pi r^2}$$

This time you **must** use the radius *r*.

For example, to find the area of the circle whose radius is 6·4 cm, the working is

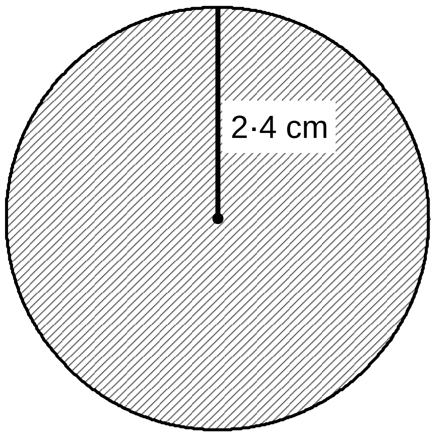


$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 6 \cdot 4^2 \\ &= 128 \cdot 68 \text{ cm}^2 \text{ to 2dp} \end{aligned}$$

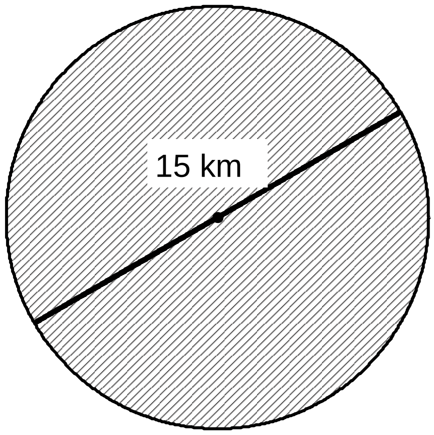


84) Calculate the full area.

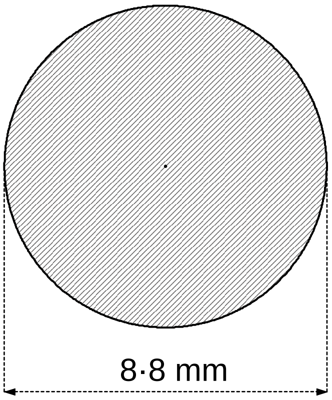
a)



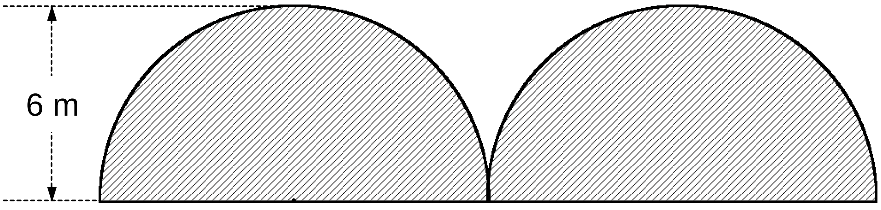
b)



c)



d)



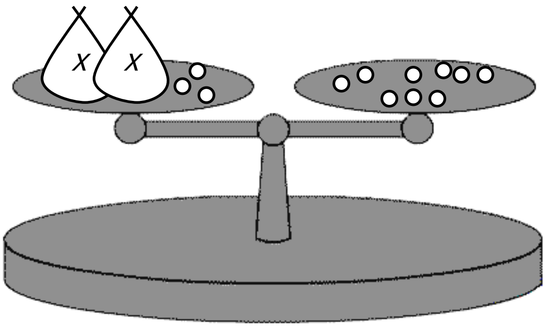
Equations

85) An **equation** is like a puzzle where you have to try and work out what number the mystery letter stands for. Equations have two sides, separated by an "=" sign.

For example,

$$2x + 3 = 9$$

We can think of this equation as a set of scales with $2x + 3$ on one side and 9 on the other.



Provided we follow the basic rule of doing exactly the same to both sides of the equation, the scales will stay balanced.

We can add, subtract, multiply or divide provided we **treat each side in exactly the same way**. For example we could, if we wanted, add 5 more weights to each side:

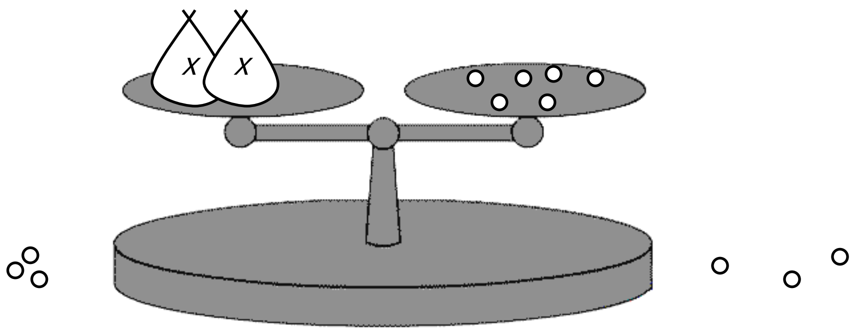
$$2x + 3 = 9$$

$$2x + 3 + \mathbf{5} = 9 + \mathbf{5}$$

$$2x + 8 = 14$$

This is still a true statement, but it doesn't help us work out the value of the mystery letter x.

The correct thing to do here is remove 3 weights from each side:



The working goes

$$2x + 3 = 9$$

$$2x + 3 - \mathbf{3} = 9 - \mathbf{3}$$

$$2x = 6$$

Now we can finish or solve the equation easily.
2 whats make 6? Divide each side by 2 to get the solution:

$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

86) Here are some examples of how you *could* lay out the working for equations:

a)

$$5d - 3 = 12$$

The $5d$ has had 3 subtracted from it so we begin by adding that back on.
We must add 3 to **both** sides!

$$5d - 3 + \mathbf{3} = 12 + \mathbf{3}$$

$$5d = 15$$

$$\frac{5d}{5} = \frac{15}{5}$$

$5d$ is 5 times the size of our mystery letter d so we must divide both sides by 5

$$d = 3$$

You don't **have** to show all these steps if you know what you are doing, but the **bare minimum** in an exam setting would be

$$5d - 3 = 12$$

$$5d = 15$$

$$d = 3$$

We can always check our answer by substituting the d value into the original equation and testing each side separately. If d is really 3 then

$$\begin{aligned}\text{LH side} &= 5d - 3 \\ &= 5(3) - 3 \\ &= 12 \\ &= \text{RH side}\end{aligned}$$

So we know our answer is correct.

b) $7h + 2 = 16$

The $7h$ has had 2 added on so we begin by removing that 2.

$$7h + 2 - \mathbf{2} = 16 - \mathbf{2}$$

$$7h = 14$$

$$\frac{7h}{7} = \frac{14}{7}$$

Now we must divide both sides bv 7

$$h = 2$$

Check the solution for yourself.

c) $3 - 2x = 7$

A nasty one.

$$3 - 2x - \mathbf{3} = 7 - \mathbf{3}$$

The x's have a normal, positive 3 sitting beside them so we can start by taking 3 off each side.

$$-2x = 4$$

Note the sign of the x's is negative. That sign cannot disappear just because we don't like it!

$$\frac{-2x}{-2} = \frac{4}{-2}$$

If we have $-2x$ we must divide both sides by -2.

$$x = -2$$

Normal 4 divided by -2 : different signs,

d) Sometimes the answer to an equation will be a fraction or a decimal.

$$7e - 2 = 8$$

$$7e - 2 + \mathbf{2} = 8 + \mathbf{2}$$

$$7e = 10$$

$$\frac{7e}{7} = \frac{10}{7}$$

$$e = \frac{10}{7} \text{ or } 1\cdot 428571...$$

In such cases the fraction is the "best" answer. Only round decimals off if asked to in the question. If the decimal is nice, such as 2·5 or 6·75, just leave it as it stands.

87) Solve these simple equations, showing your working if you want:

a) $x+4=7$

b) $k-5=8$

c) $t+8=19$

d) $r+19=21$

e) $a-19=21$

f) $b-8=12$

g) $f-0\cdot5=8\cdot5$

h) $2z=10$

i) $3w=12$

j) $6q=42$

k) $4c=6$

l) $12p=30$

m) $16g=20$

n) $4d=0$

o) $\frac{1}{2}m=10$

p) $\frac{1}{5}h=4$

88) These equations definitely need working. Note the “bare minimum” comment in q86a).

a) $3x+1=10$

b) $2y-2=2$

c) $5p+7=12$

d) $2+3a=17$

e) $4w-1=15$

f) $2x-5=4$

g) $7n+2=37$

h) $0=2e-14$

i) $9p-4=32$

j) $5+3h=11$

k) $7+5g=17$

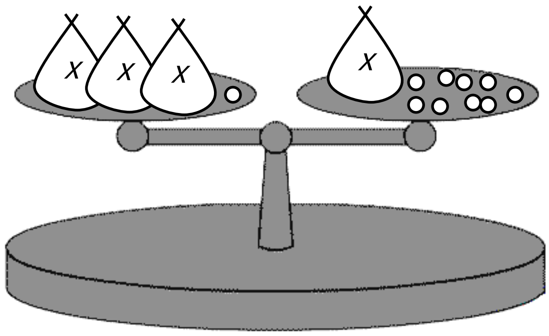
l) $70=e+66$

m) $25=d+9$

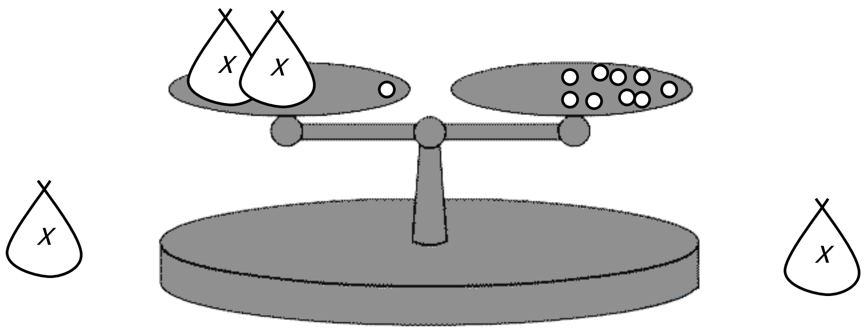
n) $81=2b-19$

89) If an equation has letters on **both** sides, the best way to start is by removing as many letters as you can from each side.

Consider the equation represented in the picture below : $3x+1=x+9$



We should try to begin by removing as many bags as we can from each side. We only have one bag on the RHS, so remove one "*x*" from each side.



$$
\begin{aligned}
3x+1&=x+9\\
3x+1-\boldsymbol{x}&=x+9-\boldsymbol{x}\\
2x+1&=9
\end{aligned}
$$

Now it is easy to finish.

$$
\begin{aligned}
2x+1&=9\\
2x+1-\mathbf{1}&=9-\mathbf{1}\\
2x&=8\\
\frac{2x}{2}&=\frac{8}{2}\\
x&=4
\end{aligned}
$$

90) Always lay out your working **down the way**. It is much easier to spot any mistakes you may have made from one line to the next.

a) $2x = x + 5$

b) $5x = 4x + 8$

c) $6y = 2y + 4$

d) $10k = 3k + 28$

e) $9r = 6r + 21$

f) $4a - 1 = 3a + 8$

g) $7w + 1 = 2w + 31$

h) $t + 1 = 6t - 14$

i) $5c + 3 = 3c + 9$

j) $5g + 3 = 8g - 21$

k) $10n + 45 = 5n + 85$

l) $3h + 11 = 5h - 15$

91) Sometimes, on one or both sides of an equation, letters have been **subtracted** from something. In that case you can simplify the equation by **adding** them back on again.

For example, in the equation $15 - 2g = g + 9$, the $2g$ on the left has been subtracted from the number 15. So you *could* begin by adding them back on:

$$15 - 2g = g + 9$$
$$15 - 2g + \boldsymbol{2g} = g + 9 + \boldsymbol{2g}$$
$$15 = 3g + 9$$
etc

Alternatively you could remove a single g from each side:

$$15 - 2g = g + 9$$
$$15 - 2g - \boldsymbol{g} = g + 9 - \boldsymbol{g}$$
$$15 - 3g = 9$$
etc

It depends whether you want the letters disappearing on the left or the right side.
Either way you should get the same final answer of $g = 2$.

If you have something like $8 - z = 20 - 3z$, which doesn't happen very often, you can either add a single z to each side making the letters vanish on the left, or add $3z$ to each side making them vanish on the right. It's up to you!

$$8 - z = 20 - 3z$$
$$8 - z + \boldsymbol{z} = 20 - 3z + \boldsymbol{z}$$
$$8 = 20 - 2z$$
etc

or

$$8 - z = 20 - 3z$$
$$8 - z + \boldsymbol{3z} = 20 - 3z + \boldsymbol{3z}$$
$$8 + 2z = 20$$
etc

92a) $20 - f = 6f + 6$

b) $33 - 2a = 3a + 3$

c) $7 - 3w = w - 1$

d) $10g = 80 - 6g$

e) $24 - 3t = 28 - 5t$

f) $60 - 3u = 40 - u$

93) Sometimes we need to solve equations which involve brackets that need expanded first.

Look at this example and see how easy this is.

$$2(x+3)=14$$

Multiply out the bracket.

$$2x+6=14$$

Subtract 6 from each side.

$$2x+6-\mathbf{6}=14-\mathbf{6}$$

$$2x=8$$

Divide both sides by 2

$$\frac{2x}{2}=\frac{8}{2}$$

$$x=4$$

Remember to check the answer to see if it makes sense:

$$\begin{aligned}2(4+3)&=2(7)\\&=14\end{aligned}$$

94) Solve these equations by first multiplying out the brackets.

a) $2(x+4)=10$

b) $3(g+1)=15$

c) $3(t-1)=21$

d) $4(d-2)=12$

e) $6(y+3)=60$

f) $7(w+2)=49$

g) $5(h-1)=45$

h) $4(t-3)=48$

i) $6(x+3)=54$

j) $44=4(7+2s)$

k) $90=6(2x-3)$

l) $66=3(7p-6)$

Change the subject of a formula

95) You have seen many formulas already in this book. Some have been "real" formulas such as $c = \pi d$, others have been made-up such as $p = \sqrt{6d + \dfrac{t^2}{u-1}}$ from q34).

All formulas have a letter on its own on the left-hand side. We call this letter the **subject** of the formula. The letters (and numbers) on the right-hand side can be thought of as the "ingredients" of the formula. If you know their values you can evaluate the subject on the left.

96) Consider the simple sum $9 = 7 + 2$. In a way, 9 is the subject of this "formula".

If we want to rearrange the three numbers to make 7 the subject instead, we have to write

$$7 = 9 - 2$$

Notice it's no longer an addition sum : formulas usually look quite different when we rearrange them.

Suppose we have a simple formula $k = f + t$ and we want to make f the subject:

$$k = f + t$$

step 1 Swap the two sides around. This gets *f* on the left.

$$f + t = k$$

step 2 We want "rid" of the *t*, which has been **added** to our *f*, so **subtract** *t* from each side.

$$f + t - \boldsymbol{t} = k - \boldsymbol{t}$$

step 3 Tidy up. The *t* appears to have "jumped" from the left side over to the right.

$$f = k - t$$

97) Another easy example : make *d* the subject of the formula $c = \pi d$.

$$c = \pi d$$

step 1 Swap the two sides around. This gets *d* on the left.

$$\pi d = c$$

step 2 We want "rid" of the π. Our *d* has been **multiplied** by π, so **divide** each side by π.

$$\frac{\pi d}{\pi} = \frac{c}{\pi}$$

step 3 Tidy up. The π appears to have "jumped" from the left side over to the right.

$$d = \frac{c}{\pi}$$

98) Rearrange these simple formulas to make the given letter the subject.

a) $h = v - 3$; v

b) $e = r + 12$; r

c) $N = P + Q$; P

d) $L = d - g$; d

e) $F = 3a$; a

f) $w = \frac{y}{7}$; y

g) $d = st$; s

h) $b = 8jx$; x

i) $C = \frac{e}{2s}$; e

j) $n = u + 3b$; u

99) Some people actually do (or teach) rearranging by jumping things over to the other side. The layout of the working is secondary, what matters is that you understand the process behind it.

Things get a little more complicated when the formula has more than one calculation in it.

For example, make *a* the subject of the formula $s = a\pi - d$.

$$s = a\pi - d$$

step 1 Swap the two sides around to get *a* on the left.

$$a\pi - d = s$$

Now comes the big decision – do we divide by π or add on *d* first?

Well, consider something similar – the equation $2x - 3 = 11$.
To solve this, you would not hesitate to begin by adding 3 to each side:

$$2x - 3 = 11$$
$$2x - 3 + \mathbf{3} = 11 + \mathbf{3}$$
$$2x = 14$$
$$\vdots$$

and you get the solution $x = 7$. So the full working is

$$s = a\pi - d$$

$$a\pi - d = s$$

$$a\pi - d + \boldsymbol{d} = s + \boldsymbol{d}$$

$$a\pi = s + d$$

$$\frac{a\pi}{\pi} = \frac{s + d}{\pi}$$

$$a = \frac{s + d}{\pi}$$

It is hard to write concise rules for rearranging, as formulas come in so many different shapes. You will build up your experience of this topic in National 5, but you should have a grasp of the basics.

100) See how many of these harder ones you can manage:

a) $q = 3n - 4$; n

b) $p = j\left(v + \pi\right)$; v

c) $w = \dfrac{b+6}{a}$; b

d) $y = mx + c$; m

e) $h = 3\left(d - z\right)$; d

f) $P = 2L + 2B$; L

g) $A = b^2$; b

h) $f = \dfrac{s}{2} + g$; s

i) $u = \dfrac{M+3}{e}$; M

j) $G = \left(x+3\right)^2$; x

Pythagoras

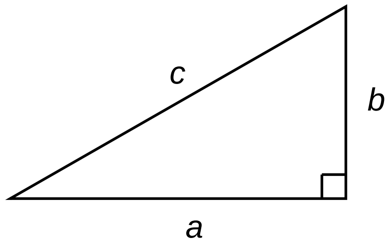
101) This famous formula works in right-angled triangles. If we know two of the sides, it allows us to calculate the length of the third side.

There are various ways to write the formula for Pythagoras Theorem:

- In words, "the square of the hypotenuse is equal to the sum of the squares of the other two sides"

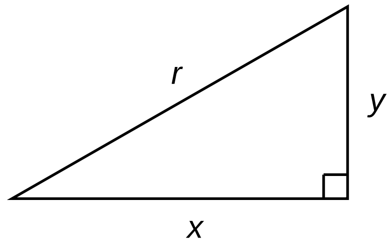
- On the National 4 formula list, it appears as

$$a^2 + b^2 = c^2$$



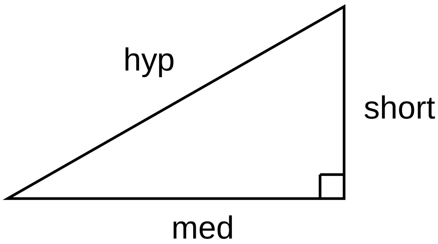
- When the author of this workbook was at school, he was taught

$$x^2 + y^2 = r^2$$



- You could also give the sides "names" and use

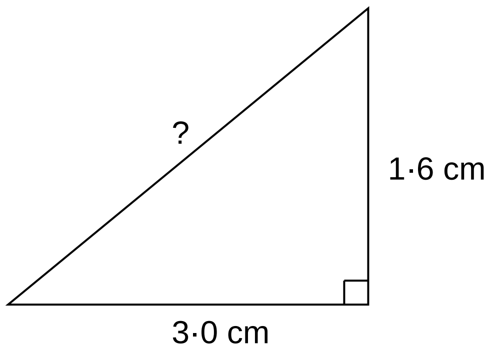
$$\text{hyp}^2 = \text{med}^2 + \text{short}^2$$



102) Here we want to work out the length of the hypotenuse.

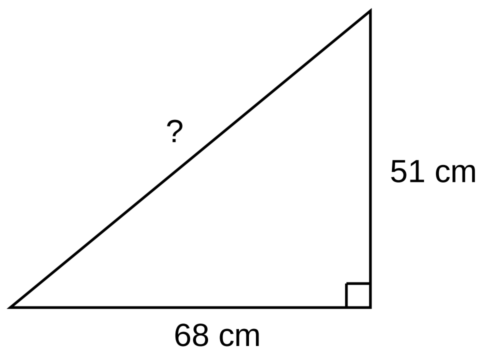
It doesn't really matter if we state the formula or not, as long as we pop the numbers in the correct places:

$$\begin{aligned} ?^2 &= 3\cdot0^2 + 1\cdot6^2 \\ &= 9 + 2\cdot56 \\ &= 11\cdot56 \\ ? &\overset{\sqrt{}}{=} 3\cdot4 \text{ cm} \end{aligned}$$

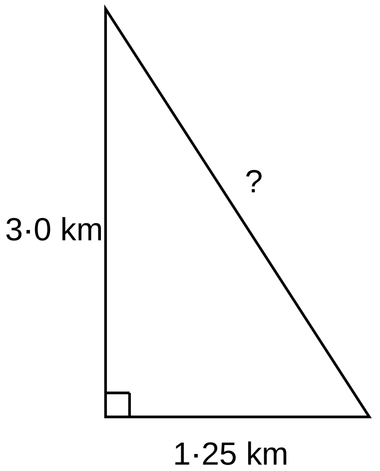


103) Calculate the length of the third side. Show your working.

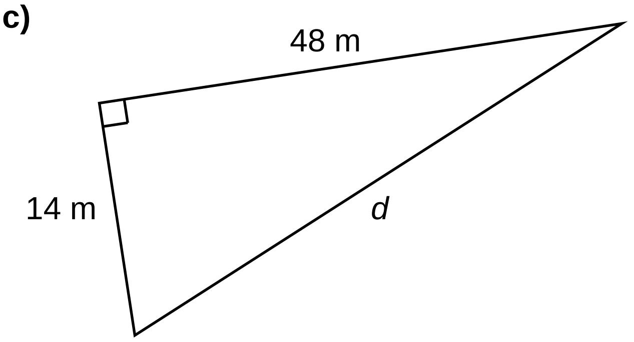
a)



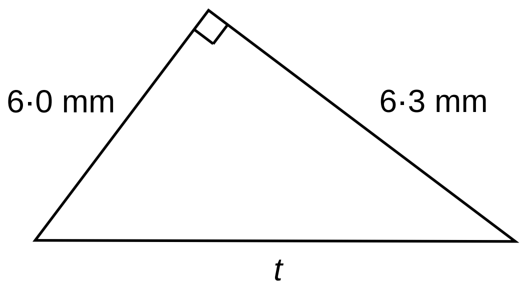
b)



c)

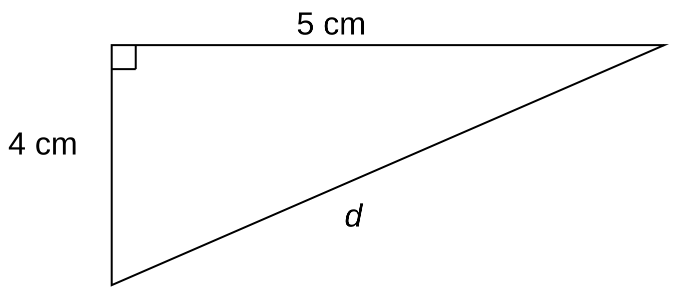


d)

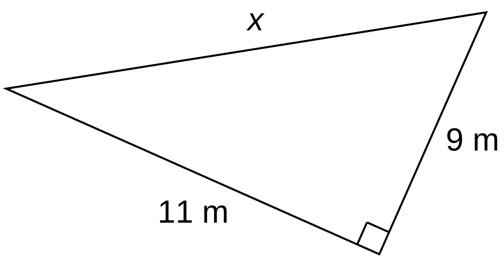


104) This time round the answers off to 1 decimal place.

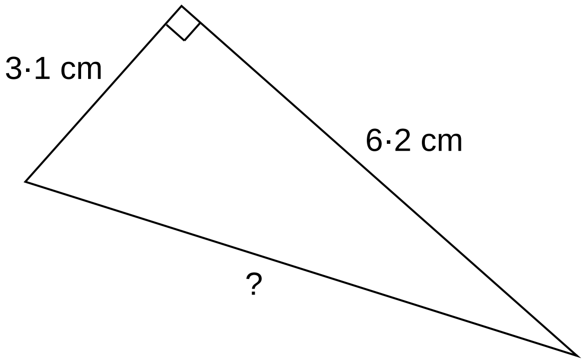
a)



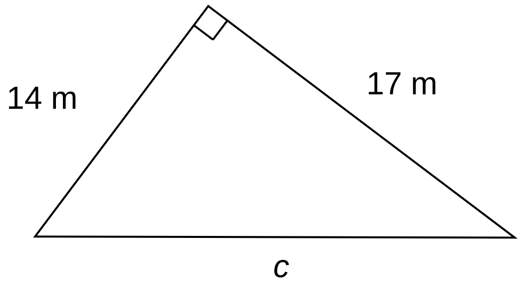
b)



c)

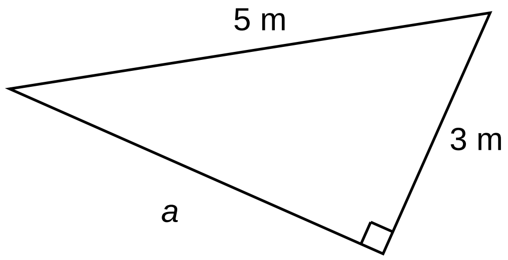


d)



105) If the missing side is one of the two **shorter** lengths, you must **subtract** the squares.

Why? Well if you try adding the squares you would get an impossible answer:



$$
\begin{aligned}
a^2 &= 5^2 + 3^2 \\
&= 25 + 9 \\
&= 34 \\
a &\overset{\sqrt{}}{=} 5\cdot 83 \text{ m}
\end{aligned}
$$

The missing side can't be 5·83 because that would be longer than the hypotenuse!

We should really be writing

$$
\begin{aligned}
5^2 &= a^2 + 3^2 \\
25 &= a^2 + 9 \\
a^2 &= 25 - 9 \\
&= 16 \\
a &\overset{\sqrt{}}{=} 4 \text{ m}
\end{aligned}
$$

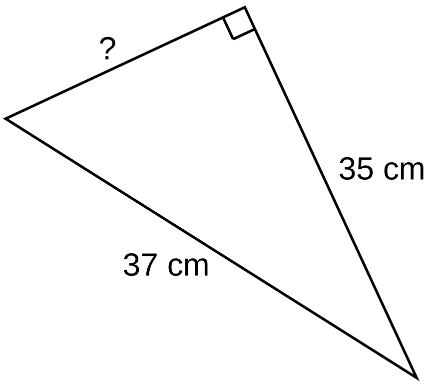
That looks more sensible! So we must subtract instead of add.

We could just write

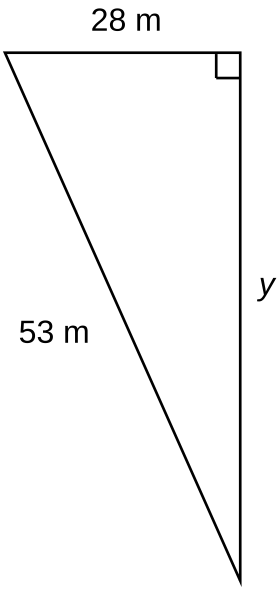
$$
\begin{aligned}
a^2 &= 5^2 - 3^2 \\
&= 25 - 9 \\
&= 16 \\
a &\overset{\sqrt{}}{=} 4 \text{ m}
\end{aligned}
$$

106) Calculate the missing lengths, checking your answers look sensible. Show your working.

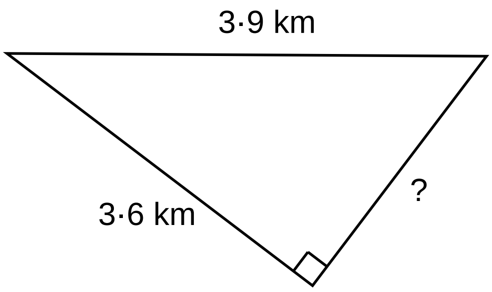
a)



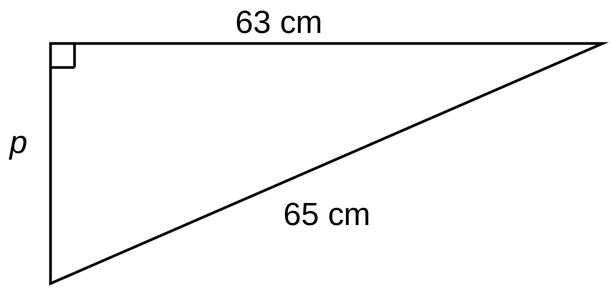
b)



c)

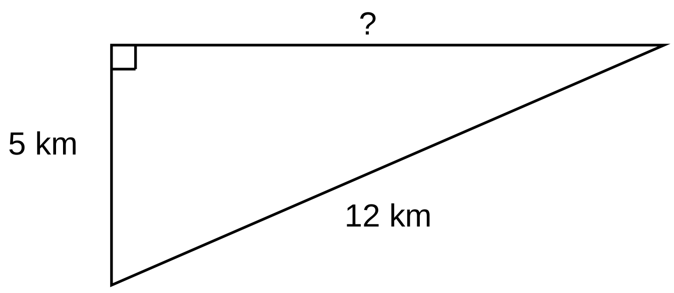


d)

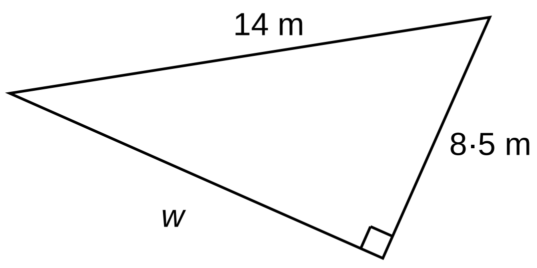


107) Round the answers off to 1 decimal place.

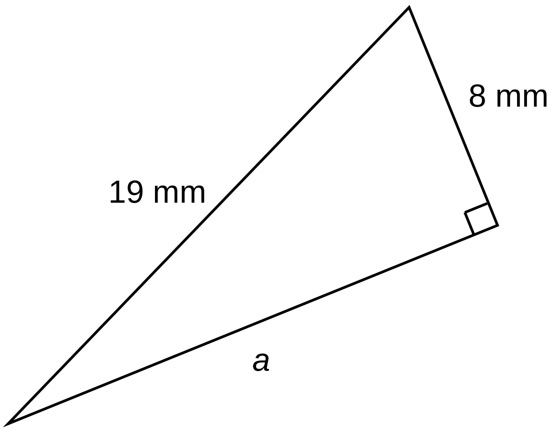
a)



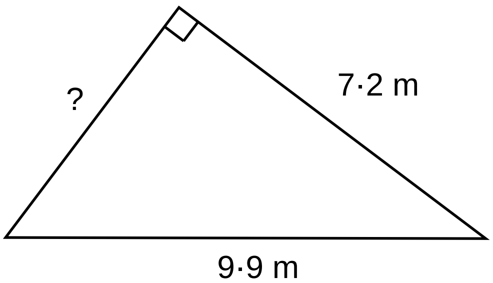
b)



c)



d)



Cross-multiplying

108) Cross-multiplying is a really useful trick for getting rid of fractions in your working.
It is used when you have **two fractions separated by an "=" sign**.

We can cross-multiply here $\frac{12}{18} = \frac{2}{r}$ ☑

but not here $\frac{5}{a} = \frac{1}{2} + a$ ☒

because there's an extra term on the right-hand side.

109) The "formula" is

$$\boxed{\text{top} \times \text{bottom} = \text{bottom} \times \text{top}}$$

(in a cross shape)

Study this example and then solve the following equations.

Solve $\frac{4}{10} = \frac{6}{w}$

$$\begin{aligned} 4 \times w &= 10 \times 6 \\ 4w &= 60 \\ w &= 15 \end{aligned}$$

a) $\frac{3}{8} = \frac{6}{t}$ **b)** $\frac{3}{4} = \frac{9}{m}$ **c)** $\frac{14}{21} = \frac{4}{k}$ **d)** $\frac{c}{6} = \frac{5}{10}$

e) $\frac{y}{12} = \frac{2}{3}$ **f)** $\frac{e}{35} = \frac{4}{10}$ **g)** $\frac{x}{15} = \frac{4}{5}$ **h)** $\frac{d}{20} = \frac{3}{10}$

i) $\frac{8}{a}=\frac{1}{2}$

j) $\frac{10}{h}=\frac{3}{9}$

k) $\frac{6}{10}=\frac{p}{25}$

110) Sometimes you have a whole number on one side instead of a fraction. You can easily make the whole number into a fraction by putting it over a "1", then you can cross-multiply.

Rewrite then solve by cross-multiplying. The first one has been started for you.

a) $\frac{b}{5}=3$

$\frac{b}{5}=\frac{3}{1}$

b) $\frac{p}{4}=7$

c) $\frac{d}{16}=2$

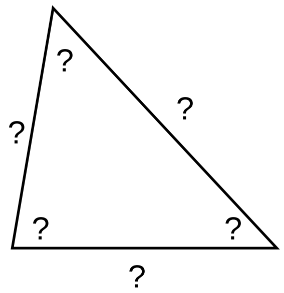
e) $\frac{24}{k}=8$

f) $\frac{n}{0\cdot625}=8$

d) $\frac{9}{y}=3$

Trigonometry

111) There are six measurements we can make in a triangle – the lengths of the three sides and the sizes of the three angles. Trigonometry is about the connections between the lengths and the angles.

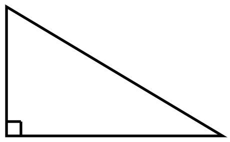


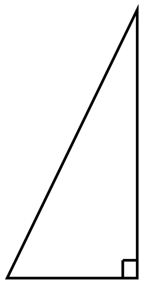
We often know three or four of these six figures. In Trig we learn how to use the figures we already know to work out the ones we don’t.

112) At National 4, we restrict ourselves to looking at right-angled triangles.

The longest side in any right-angled triangle has a special name. We call it the ______________.

It is always situated across from the right-angle.
Label the hypotenuse on each triangle.





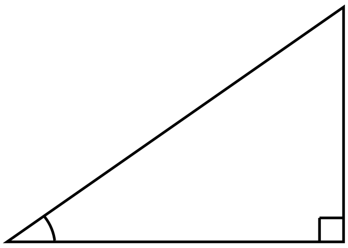
113) The other two sides have labels too, “**Opposite**” and “**Adjacent**”.
However their position can vary.

In all Trig diagrams, we will be interested in one of the other two angles. We might be given its size or we might be asked to calculate its size, but a second angle is always involved. In the following diagrams it has been marked with a curve.

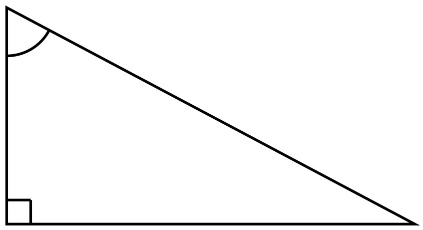
The side opposite this angle gets labelled _______________ and the remaining side is labelled _______________

Fill in all three labels on the following triangles.

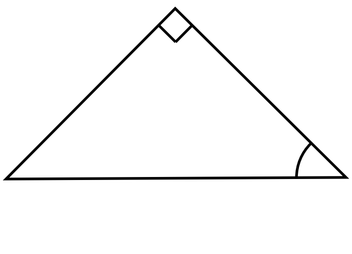
a)



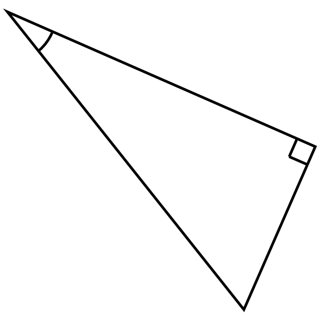
b)



c)



d)

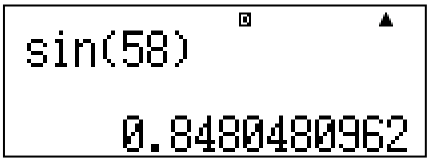


Before doing any more of this booklet, make sure your calculator is in Degree mode. There should be a little “D” at the top of your screen. On the Casios, type [SHIFT] [MODE] [3].

114) Every angle there is, such as 27° or 81·5°, has three important numbers associated with it.

These numbers are called the angle’s **Sine**, **Cosine** and **Tangent**, and are usually decimals. Your calculator is able to tell you them.

Suppose we have an angle of 58°. To obtain its sine on our Casio calculators, type in [sin] [5] [8] [=]. You can close the brackets round the angle if you like being neat.



Write down the decimals, rounded to 3 decimal places, for sine, cosine and tangent of 58°.

$\sin 58^\circ =$

$\cos 58^\circ =$

$\tan 58^\circ =$

115) Look up these, again rounding to 3 decimal places if necessary:

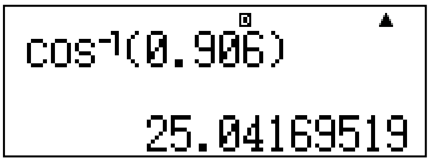
a) $\sin 11^\circ =$ **b)** $\sin 72^\circ =$ **c)** $\cos 40^\circ =$

d) $\tan 29^\circ =$ **e)** $\cos 82\cdot 1^\circ =$ **f)** $\sin 30^\circ =$

g) $\tan 45^\circ =$ **h)** $\cos 30\cdot 5^\circ =$ **i)** $\tan 83^\circ =$

116) Often in Trig we are **told** the decimal and want to know the angle it corresponds to. For example, what angle has a cosine of 0·906?

To retrieve the angle from the decimal, we need to go above the [cos] button where it says $(\cos^{-1})$. To access this you need to press the top-left button on your calculator. On the Casios, type [SHIFT] [cos].



Please check you agree with the answer shown.
We would usually round an angle like this off to one decimal place, 25·0°.

117) What angle, correct to 1 decimal place, has a

a) sine equal to 0·218?

b) tangent equal to 1·2?

c) cosine equal to 0·35?

d) sine equal to $\frac{1}{4}$?

118) A shorter way of writing "the angle whose sine is equal to 0·822" is

$$\sin^{-1} 0\cdot 822$$

Work out these angles, again to 1dp

a) $\sin^{-1} 0\cdot 822 =$

b) $\tan^{-1} 0\cdot 822 =$

c) $\cos^{-1} 0\cdot 3 =$

d) $\sin^{-1}\left(\frac{1}{5}\right) =$

119) A mixture. Give at least the first three decimal places of sin/cos/tan decimals, and round angles to 1 dp.

a) the cosine of 13°

b) the sine of 45°

c) the angle whose tangent is 0·6

d) the angle whose sine is 0·316

e) $\tan 20^\circ$

f) $\sin^{-1}\left(\frac{2}{3}\right)$

g) $\tan^{-1}\left(2\frac{1}{2}\right)$

h) $\cos 78\cdot 8^\circ$

120) We could easily measure the lengths in any triangle if we wanted. The decimals for sine, cosine and tangent simply come from dividing one length by another:

$$\text{Sine of an angle} = \frac{\text{length of Opposite side}}{\text{length of Hypotenuse}}$$

$$\text{Cosine of an angle} = \frac{\text{length of Adjacent side}}{\text{length of Hypotenuse}}$$

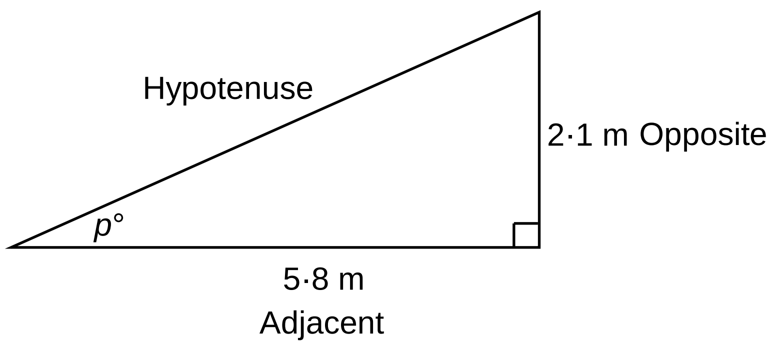
$$\text{Tangent of an angle} = \frac{\text{length of Opposite side}}{\text{length of Adjacent side}}$$

These formulas are a bit of a mouthful, so we shorten them to the word

SOH CAH TOA

This is our three-in-one Trig formula. We only ever use one part of it at a time though.

120) Let’s use SOH CAH TOA to calculate the size in degrees of angle $p°$ in this triangle. The sides have been labelled with the correct words relative to angle $p°$.



Now we need to decide which part of the three-in-one formula to use.
Write out SOH CAH TOA and tick the sides we know the lengths of.
Here it’s the Opposite and the Adjacent:

✓ ✓ ✓✓
SOH CAH TOA

This narrows it down to the Tan part.

✓ ✓ ✓✓
SOH CAH TOA

So we now have

$$\tan p^\circ = \frac{\text{Opp}}{\text{Adj}}$$

Now pop the known lengths in

$$\tan p^\circ = \frac{2\cdot 1}{5\cdot 8}$$

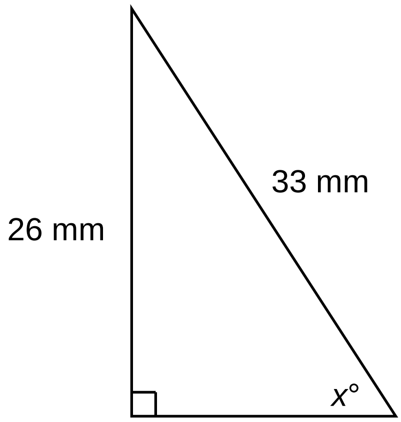
and divide to obtain a decimal.

$$\tan p^\circ = 0\cdot 362...$$

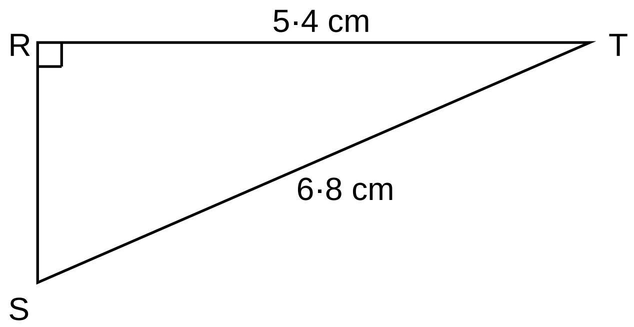
Now we retrieve the angle from the decimal as in q117):

$$p = \tan^{-1} 0\cdot 362...$$
$$p = \underline{\hspace{2cm}}^\circ$$

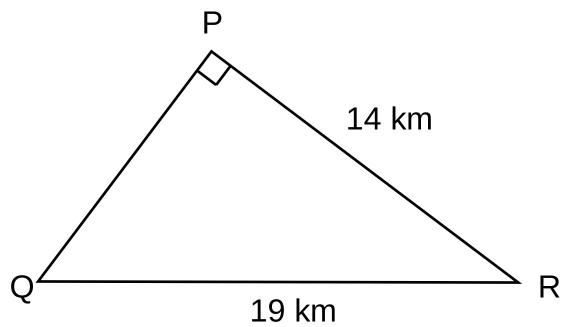
122) Begin by labelling the sides of this triangle then calculate the size in degrees of angle $x°$.



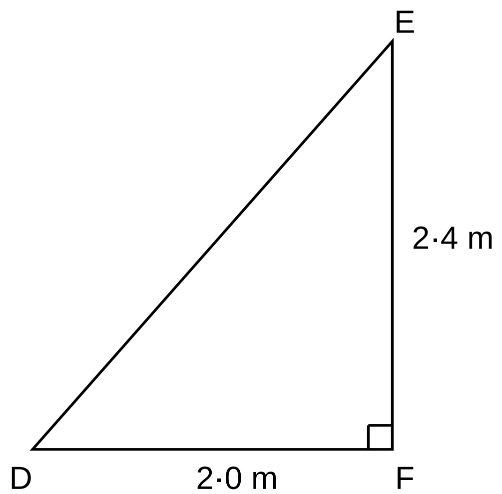
123) Calculate the size in degrees of angle T.



124) Calculate the size in degrees of angle Q.

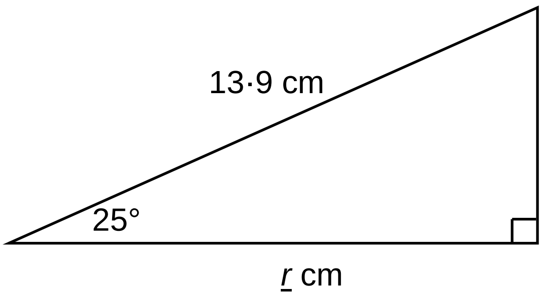


125) Calculate the size in degrees of angle D.



126) As well as using SOH CAH TOA to find unknown angles, we can also use it to calculate missing lengths.

In this triangle we want to work out the distance *r* cm.
As usual we are told three measurements, but this time it's two angles and a length.



We begin the working in the usual manner by labelling the sides and writing down our formula SOH CAH TOA.

This time we can only tick the Hypotenuse, but we're trying to work out the Adjacent side "*r*" so we should write a "?" above the A's.

$$\begin{array}{c} \quad\checkmark \quad ?\checkmark \quad\quad ? \\ \text{SOH CAH TOA} \end{array}$$

This narrows it down to the Cos part.

$$\begin{array}{c} \quad\checkmark \quad ?\checkmark \quad\quad ? \\ \text{SOH CAH TOA} \end{array}$$

So

$$\cos 25^\circ = \frac{\text{Adj}}{\text{Hyp}}$$

As before, pop the numbers in. This time we know the size of the angle, and therefore its cosine is just a decimal. Your calculator will tell you it.

$$0 \cdot 906... = \frac{r}{13 \cdot 9}$$

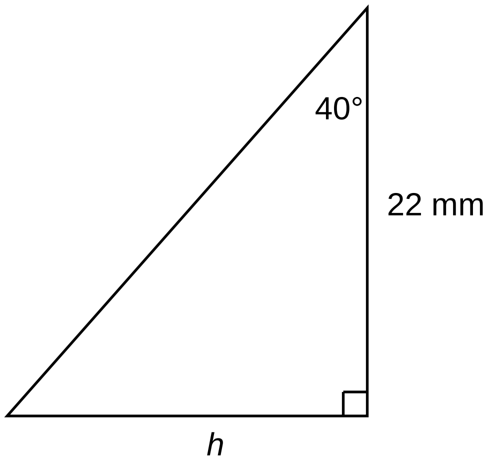
Now we use cross-multiplying as in q110) to get rid of the fraction.

$$\frac{0 \cdot 906...}{1} = \frac{r}{13 \cdot 9}$$

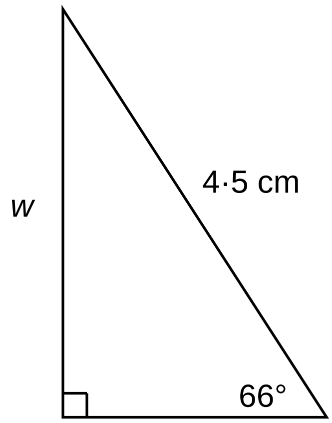
$$r = 0 \cdot 906... \times 13 \cdot 9$$
$$r = \underline{\qquad\quad} \text{ cm to 1 dp}$$

127) Use similar working to calculate these missing lengths.

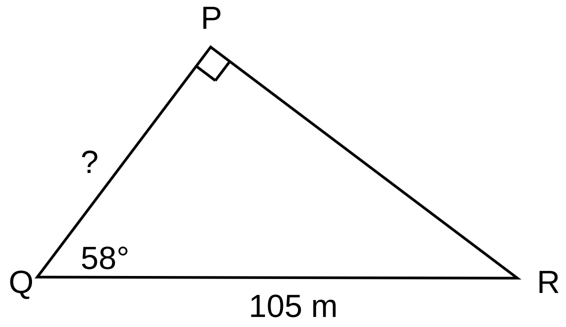
a)



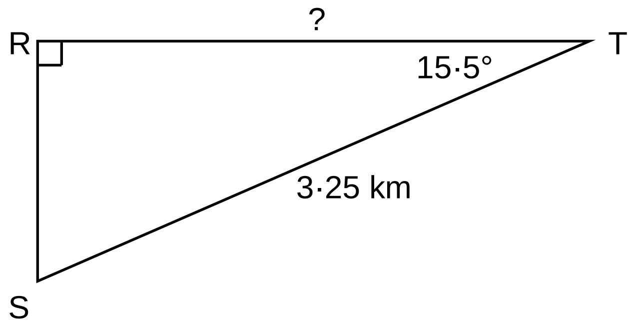
b)



c)



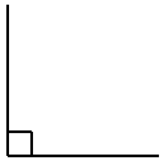
d)



Angle properties

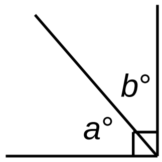
128) You need to know various facts and vocabulary about angles.

A right-angle is 90°



Two angles which add up to 90°are said to be **complementary**.

a) Calculate the value of *b* if $a = 53$.

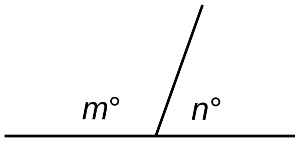


b) Calculate the **complement** of 82°.

129) A straight angle is 180°

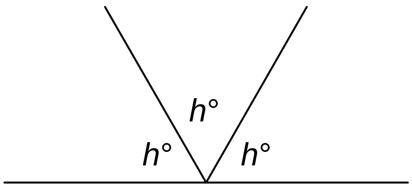
Two angles which add up to 180°are said to be **supplementary**.

a) Calculate the value of *n* if $m = 115$.

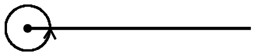


b) Calculate the **supplement** of 31°.

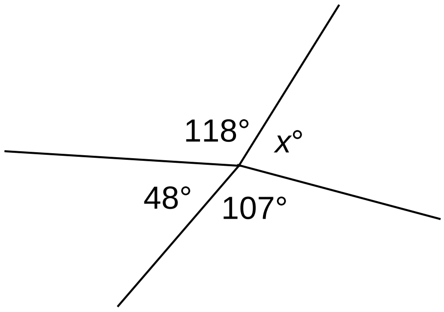
c) Calculate the value of *h*.



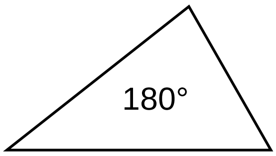
130) A full turn is 360°



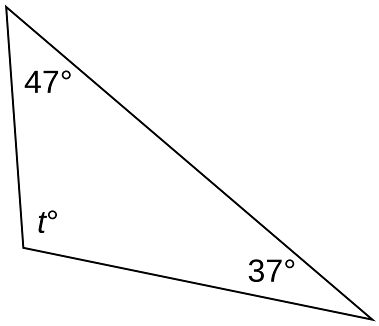
Calculate the value of *x*.



131) The angles in any triangle add up to 180°

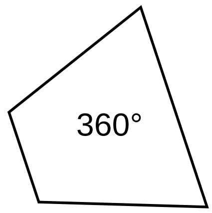


a) Calculate the value of *t*.

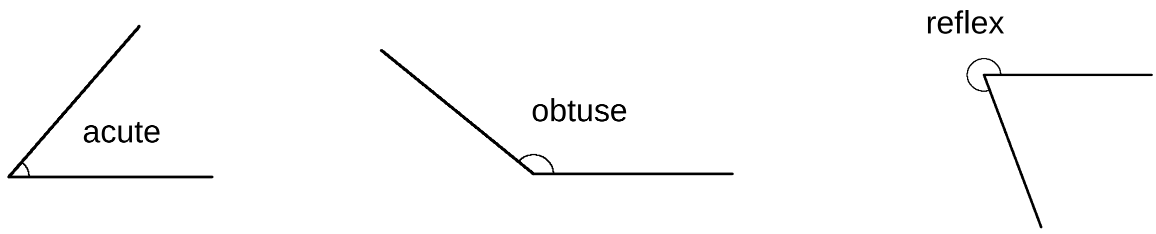


b) "Bars" on lines in a diagram indicate equal lengths. Calculate the required angles.

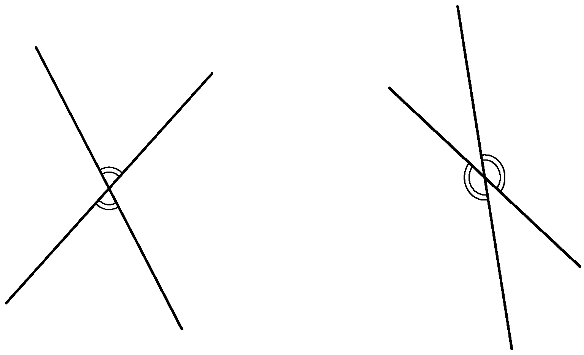
132) The angles in any quadrilateral add up to 360°



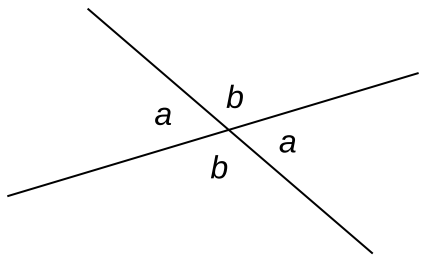
133) Types of angle:



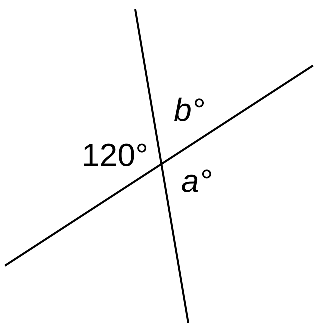
134) Opposite pairs are equal in an X-shape. Sometimes called **vertically opposite** angles.



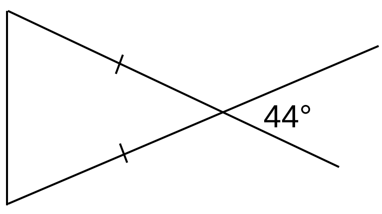
Adjacent angles are supplementary in this situation, in other words $a + b = 180$



a) Evaluate *a* and *b*.

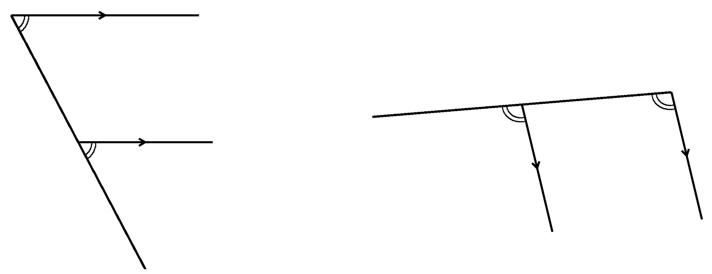


b) Fill in all the angles.

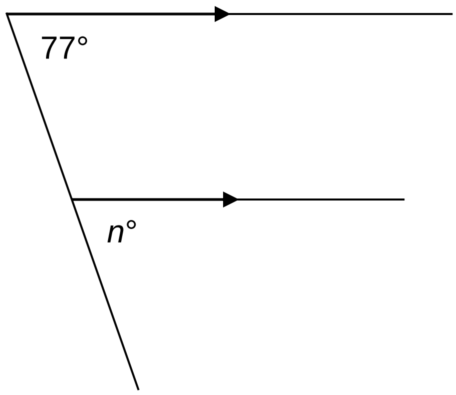


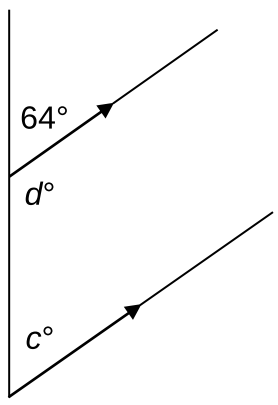
135) The next three diagrams all feature parallel lines. On a diagram, we show lines are parallel by marking them with arrows.

F-angles are equal. Sometimes called **corresponding** angles.

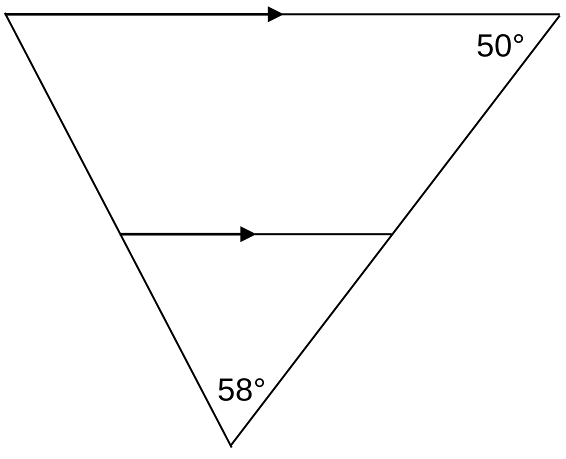


a) Work out the sizes of the lettered angles.

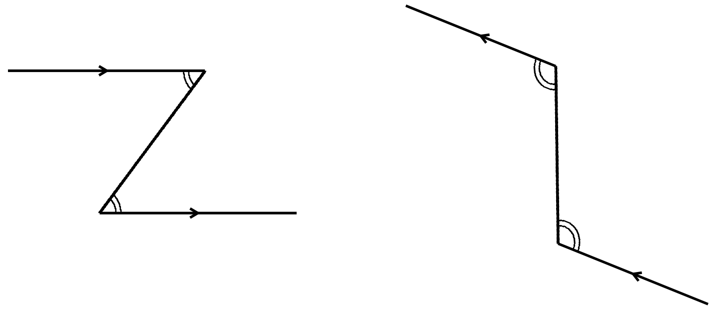




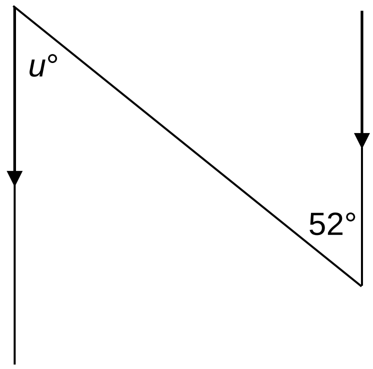
b) Fill in the sizes of **all** the angles.

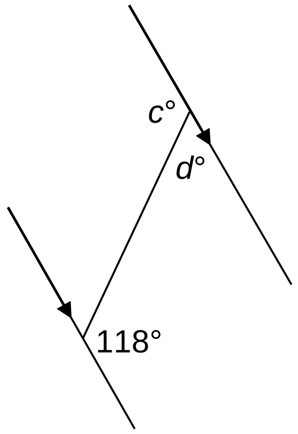


136) Z-angles are equal. Sometimes called **alternate** angles.

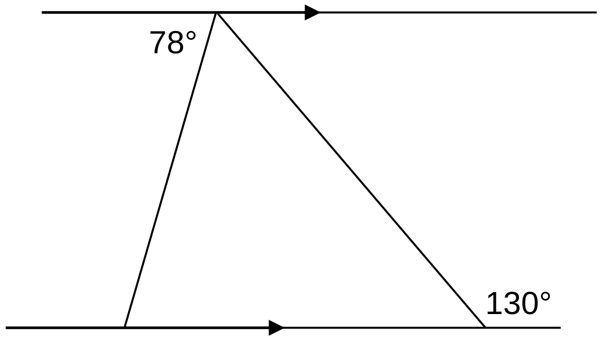


a) Work out the sizes of the lettered angles.

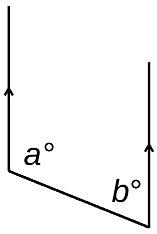




b) Fill in the sizes of **all** the angles.

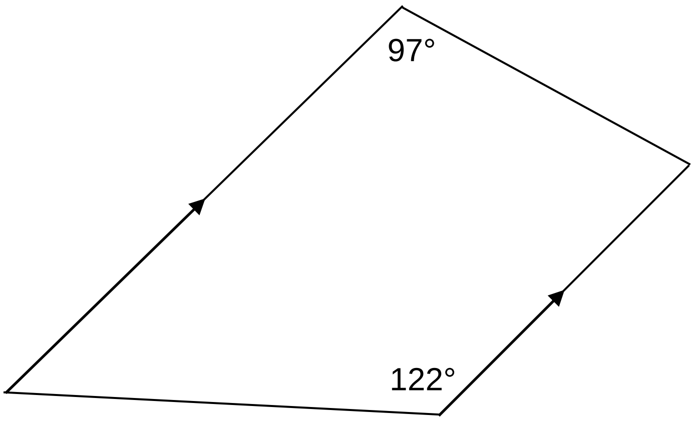


137) U-angles add up to 180°. They are found in parallelograms.



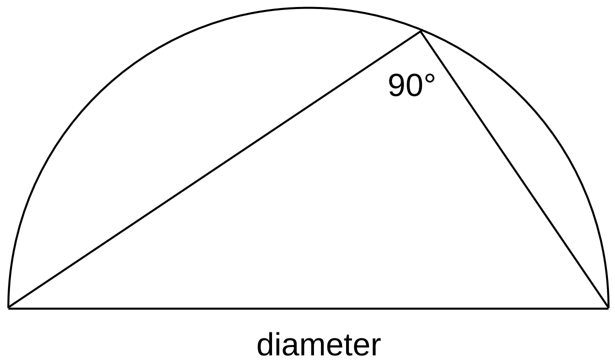
$a + b = 180$

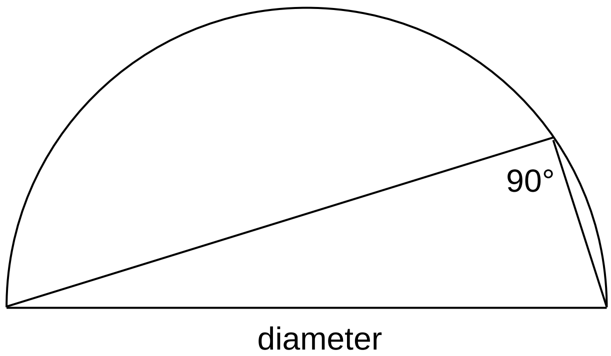
Fill in the sizes of all the angles. Check your total using q132).



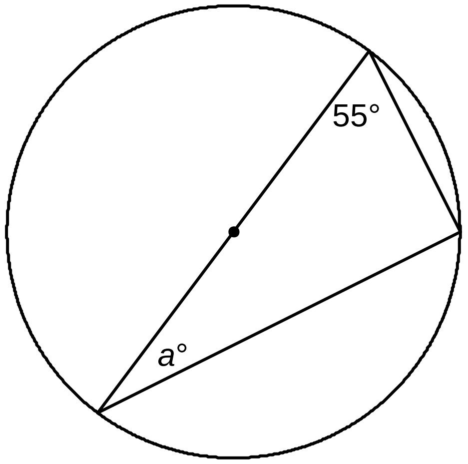
138) Diagrams containing circles often have "hidden" right angles in them. In other words, angles which are 90° but are not marked as such. You need to be able to recognise them.

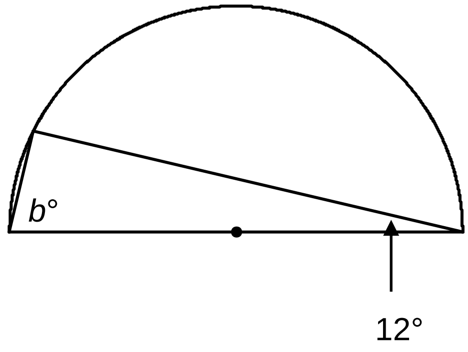
The angle in a semicircle is always 90°. The three angles in the triangle add up to 180° of course, so if you know one of the other angles you know all three.

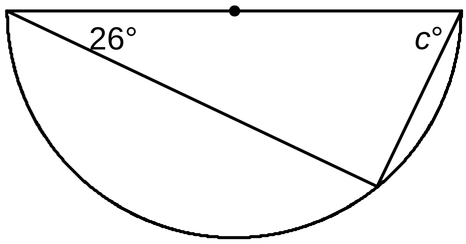


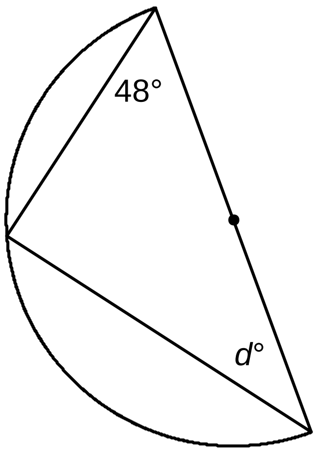


139) The following diagrams feature diameters or semicircles.
Calculate the size of the marked angles.

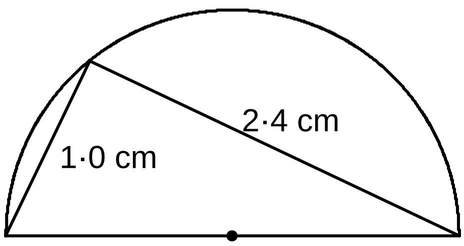






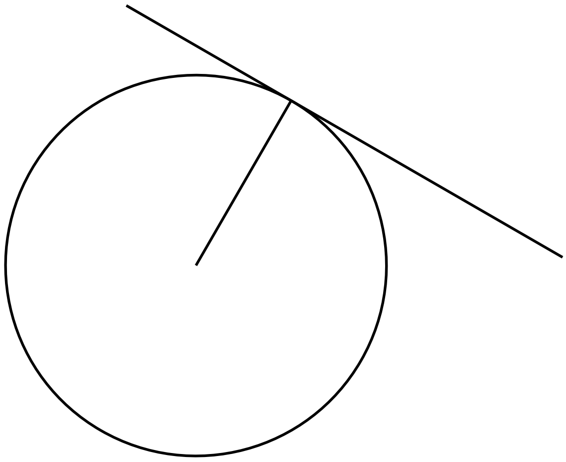


140) Such triangles are right-angled, so we can do Pythagoras in them.
Calculate the diameter of this semicircle.

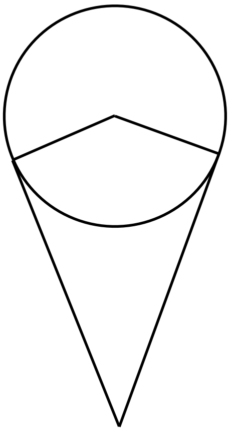


141) A tangent is a line which just touches a circle at one point.

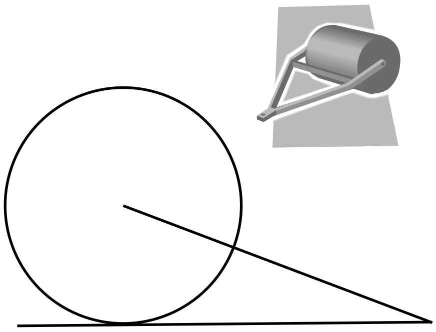
You also get “unmarked” right angles between the radius and the tangent at the point of contact.



Typical diagrams containing this might be

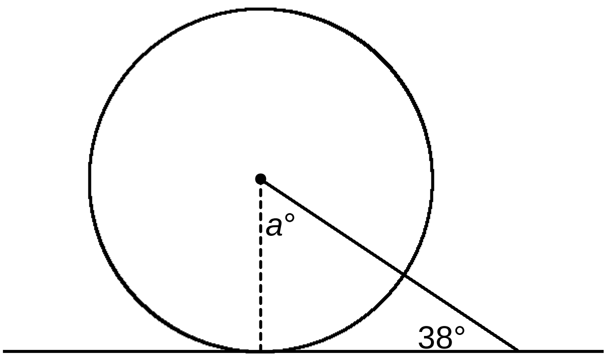


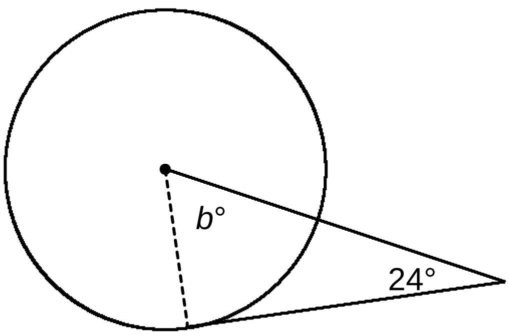
a “tangent kite”

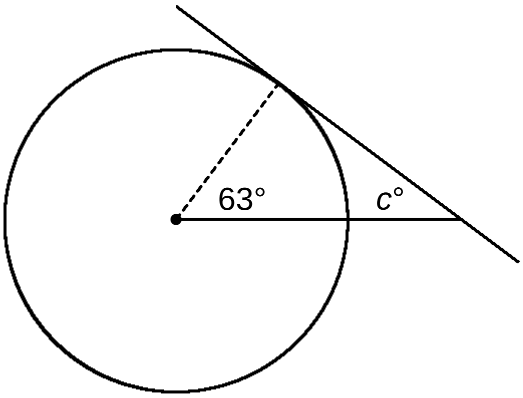


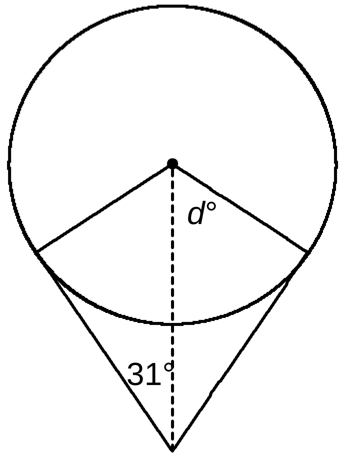
a roller on the ground

142) The following diagrams feature tangents. Calculate the size of the marked angles.









143) The tangent PQ touches the circle, centre O, at T.

Calculate the size of angle MOT.